Polynomial Law of Sensation

SUE DOE NIHM *Chang Ri Law University*

ABSTRACT: *A new theory proposes that sensation grows as a polynomial function of physical intensity. The theory reproduces all of the published data perfectly without error. The degree of the polynomial is independent of whether category ratings or magnitude estimations are used as the dependent variable; it is independent of stimulus range, number of categories, value of the standard, first stimulus, modulus, stimulus spacing, and all other contextual features of the experiment except the number of stimuli. Because the polynomial law always provides a superior fit to the data, it should supersede the logarithmic and power laws of sensation.*

Judgments of the sensory magnitude of stimuli varying in one dimension of physical intensity have often been fit by functions of the form:

$$R = a\,(\phi - \phi_0)^k + b, \qquad (1)$$

where R is the subjective estimate of magnitude, ϕ is the physical intensity, and a, ϕ_0, k, and b are constants. One attractive feature of Equation 1 is that it can be used to fit a large variety of curves and will give a reasonable approximation to almost any set of data that might reasonably be obtained in psychophysical experiments.

Another attractive feature of Equation 1 is that although additional parameters are estimated, the exponent, k, is the only one that need be reported or discussed. It is also common practice to report only the exponent and not the data from which it was calculated. Thus, the exponent is a convenient number which represents the data so that investigators can discuss this single value and need not explain the data.

The most important use of the power function is as a basis for deciding among substantive theories of sensory processing and information integration. Experimental procedures that yield a good fit to the power function are preferred to procedures yielding data that are inconsistent with the power function. Psychologists use the assumption of a power function to test alternative theories of sensation, information integration, and judgment.

In spite of these advantages, however, there are drawbacks to the power function. First, the data are never fit exactly by the function. There are always systematic deviations. Second, the exponent depends on whether ratings or magnitude estimations are the dependent variable, on the range of the stimuli, on the value of the standard (if any), and on a variety of other experimental conditions. It also depends on the statistical estimation procedures used to fit it. The same data can lead to different exponents, and different data can lead to the same exponent.

Consequently, during my stay in the United States, I have fit a new theory to all of the psychophysical data published to date. The new theory fits all of these data perfectly without error. Furthermore, the new theory provides an index of the data that is independent of the experimental details.

I have therefore concluded that sensation grows not as the log, not as a power, but instead as a polynomial function of stimulus intensity:

$$R = a_0 + a_1\phi + a_2\,\phi^2 + a_3\phi^3 + \ldots + a_k\phi^k, \quad (2)$$

where R is the sensation, ϕ is the physical value, a_0, a_1, a_2, a_3, . . . a_k are constants, and k is the degree of the polynomial. Although there are many constants to estimate, it is proposed that the degree is the only one of scientific importance, hence the only one that need be reported.

A great advantage of the polynomial theory is that the powers of the physical variables are always integers and the degree is always an integer, whereas power function exponents are often fractions. Thus, the polynomial law is more like the laws of physics, which are often polynomials with integral powers. For example, the height of an object dropped from a high place will be given by

I wish to express gratitude to Michael H. Birnbaum, who during my visit to the United States provided me with the use of his computer account (required to analyze all of the previously published psychophysical data) and who has agreed to handle requests for reprints and correspondence, which should be sent to Michael H. Birnbaum, Department of Psychology, University of Illinois, Champaign, Illinois 61820.

the equation: $H = a_0 + a_1 t + a_2 t^2$, where H is height, t is time, and a_0, a_1, and a_2 are constants. It should be clear that the coefficients will depend on trivial details such as the starting height, starting time, and the gravitational constants, which are analogous to experimental context variables in psychophysics. Thus, the coefficients convey little of general scientific value. What counts is the degree of the polynomial. To express the law, it suffices to say that height is a quadratic function of time.

There are two other important advantages of the polynomial theory. First, I have discovered that sensation is *always* a polynomial function of intensity. Correlations between the theory and the data are always 1.000; errors are always exactly zero.[1] Second, I have found that the degree of the polynomial is always independent of all of the experimental manipulations affecting the power function exponent (except one noted below). It is also independent of whether the dependent variable is category ratings or magnitude estimations, of the stimulus range, the standard, the modulus, the first stimulus, etc., etc.

One important finding has emerged in the analysis of previous psychophysical data: The degree of the polynomial is always one less than the number of stimuli. This exciting result would never have been discovered using power functions. It seems the most exciting result discovered since Fechner's law. When I return to my country, I hope to obtain a grant to investigate the limits of the applicability of the polynomial law and of the number of stimuli effect, which I have christened "Nihm's Law."

Finally, there are a couple of problems that should be mentioned. Several colleagues, who I am certain did not intend to hurt my feelings, have criticized Nihm's Law. They say that it is merely "curve fitting" and not even a scientific theory. I must admit that it is difficult to conceive of a disproof, since sensation is always a polynomial function of intensity. I do, of course, intend to continue to test the law to see if it holds up. My critics also say that just because sensations can be *fit* perfectly by a polynomial function, it does not mean that sensations *are* a polynomial function of stimulus intensity. But these criticisms seem also to apply to the power law and the other "laws" of psychophysics. Perhaps I have missed something because of my limited English, but I have read no theoretical justification of the power law that rules out the polynomial theory. Apparently, the only justification for any psychophysical "law" is that certain functions give approximations to certain types of data in certain conditions. But the polynomial always gives a perfect fit. Consequently, *unless an experiment can be proposed that will disprove the polynomial law* in favor of another law such as the power law, I shall continue to believe that sensation grows as a polynomial function of intensity.

In future investigations, the polynomial law should be used to determine the status of experimental methods and as a validity criterion for testing substantive theories.

[1] In those instances in which the authors reported only the exponent, it had to be assumed that the data were perfectly compatible with the power function. Still, the polynomial law gave a perfect fit to the data.