Empirical Evaluation of Third-Generation Prospect Theory

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**Abstract**

Third generation prospect theory (Schmidt, Starmer, & Sugden, 2008) is a theory of choices and of judgments of highest buying and lowest selling prices of risky prospects. Highest buying and lowest selling prices are also called willingness to pay (WTP) and willingness to accept (WTA), and the gap between them is sometimes called the “endowment effect” and was previously called the “point of view” effect. Third generation prospect theory (TGPT) combines cumulative prospect theory for risky prospects with the theory that judged values are based on the integration of price paid or price received with the consequences of gambles. TGPT was developed independently of similar developments by Birnbaum and Zimmermann (1998) and Luce (2000). This paper reviews theoretical and empirical findings, to show that TGPT fails as a descriptive model of both choices and judgments. The data refute two theories of loss aversion, but they are consistent with configural weight models (Birnbaum & Stegner, 1979).

 Schmidt, Starmer, & Sugden (2008) proposed third-generation prospect theory (TGPT) as a unified theory to account for judgments of value of risky prospects as well as choices between such prospects. This theory was intended to account for the discrepancy between willingness to pay (WTP) and willingness to accept (WTA) and for preference reversals between choices and judgments of value.

Original prospect theory (Kahneman & Tversky, 1979) made use of a subjectively weighted utility formulation similar to that of Edwards (1953). Cumulative prospect theory (CPT) by Tversky and Kahneman (1992) is a variant of rank-and sign-dependent utility (RSDU), by Luce & Fishbein (1991), with particular functions specified. Schmidt, et al. (2008) refer to CPT as “second generation” prospect theory. Schmidt, et al. retained CPT for choices between risky prospects, but added some new assumptions to account for judgments of value such as maximal buying prices and minimal selling prices.

Birnbaum and Stegner (1979) presented a configural weight model to account for the discrepancy between judgments of highest buying and lowest selling prices. The highest buying price is also known as the willingness to pay (WTP) value, and the lowest selling price is also called the willingness to accept (WTA). Birnbaum and Stegner referred to the differences as the effects of the “judge’s point of view,” and presented a configural weight averaging model in which the configural weights of lower or higher values are affected by point of view. Thaler (1980) did not cite the earlier work, proposed the term, “endowment effect,” and suggested that the phenomena might relate to “loss aversion,” interpreted by Kahneman and Tversky (1979) as a property of the utility function—“losses loom larger than gains”. The phenomenon is sometimes viewed as a special case of a “status quo” bias (Samuelson & Zeckhauser, 1988). Tversky and Kahneman (1991) elaborated the idea that the discrepancy between highest buying price and lowest selling price for riskless goods could be explained by a utility function in which a loss of *x* has greater negative utility than that of a comparable absolute gain.

To represent risky gambles, Schmidt, et al. (2008) proposed TGPT, which used CPT, combined with the assumption that prices paid or accepted are integrated into the consequences of a prospect. In TGPT, buying or selling prices of risky prospects are theorized as decisions among mixed gambles that are affected by loss aversion, even though all of the consequences are strictly positive.

These main ideas of TGPT had already been proposed and evaluated by Birnbaum and Zimmermann (1998, Appendices), along with other theories of the so-called endowment effect. They rejected this model as a descriptive account of judgments of highest buying and lowest selling prices, as well as theories of Tversky and Kahneman (1991) and an anchoring and adjustment model.

In response to unpublished findings by Birnbaum and Yeary (1998), Luce (2000) developed a more elaborate theory in which prices and consequences are integrated via a joint receipt operation; this theory was further developed and evaluated by Birnbaum, Yeary, Luce, & Zhao (2002, in press). The main difference between TGPT and Luce’s (2000) approach is that in Luce’s approach prices are integrated with consequences via a joint receipt operation rather than by simple addition or subtraction as in Birnbaum and Zimmermann (1998) and by Schmidt, et al. (2008). In Luce’s (2000) approach, like that of Birnbaum and Stegner (1979), the utility of negative consequences (sometimes called “loss aversion”) plays no role in the theory.

The present article presents new analyses of previously published data to evaluate the empirical status of TGPT. Empirical results show that TGPT is not an accurate empirical description of either judgments of value (WTP and WTA) or of choices between prospects. The rest of this paper is organized as follows: Section 1 presents the key ideas of TGPT and presents theorems of TGPT that can be evaluated empirically; Section 2 presents evidence that theorems of TGPT are violated systematically by empirical findings. Section 3 reviews arguments against the loss aversion theory of Tversky and Kahneman (1991) for the endowment effect and it summarizes the case against CPT as a model of choices between risky prospects. Section 4 presents a configural weight model (Birnbaum and Stegner, 1979) and shows that it provides a better fit to the data and explains the major phenomena that refute TGPT; Section 5 discusses the implications of the empirical results for theories of choice and judgment.

**1. Third-Generation Prospect Theory**

 Let *G* = (*y, p; x*) represent a binary gamble to win *y* with probability *p* and otherwise to receive *x.* CPT (Tversky & Kahneman, 1992) can be written for such gambles as follows:

(1) CPT(*y, p; x*) = *u*(*y*)*W*(*p*) + *u*(*x*)[1 – *W*(*p*)] if *y ≥ x ≥* 0

CPT(*y, p; x*) = *u*(*y*)*W*(*p*) + *u*(*x*)*W*–(1 – *p*) if *y ≥* 0 *≥ x*

CPT(*y, p; x*) = *u*(*x*)*W*–(1 – *p*) + *u*(*y*)[1 – *W*–(1 – *p*)] if 0 *≥ y ≥ x*

Where CPT(*y, p; x*) is the value of the gamble; *u*(*x*) and *u*(*y*) are the utilities of the consequences; *W*(*p*) and *W*–(*p*) are decumulative weighting functions of probabilities to win or lose, respectively, where *W*(0) = *W*–(0) = 0 and *W*(1) = *W*–(1) = 1. It is assumed that a person would choose *G* over *F* if and only if CPT(*G*) > CPT(*F*).

 Equation 1 (CPT) is called “second-generation” prospect theory (Schmidt, et al., 2008). In third-generation prospect theory (TGPT), CPT is retained for choice, and it is further assumed that the decision maker integrates the price of a prospect with the prizes when evaluating decisions to buy or sell. In willingness to pay for *G* = (*y, p; x*), where *y* > *x* > 0, it is assumed that the subject considers that if he/she pays B and wins *y*, then the gain will be *y* – B, but if the gamble yields only *x*, then the loss will be *x* – B. Similarly, in willingness to accept, it is assumed that the subject considers a sale for S to be a gain when *x* occurs, since the profit is S – *x*; but the seller considers it a loss if the higher outcome *y* occurs, because the seller would have been better off to have kept the gamble, so seller experiences a loss of S – *y*. Thus, buying and selling prospects with strictly positive consequences involve the evaluation of mixed gambles, and psychological losses invoke “loss aversion.”

These assumptions of TGPT (Schmidt, et al., 2008, p. 209) lead to the following expressions for B (WTP) and S (WTA):

(2) CPT(*y – B, p; x – B*) = 0

(3) CPT(*S – y, p; S – x*) = 0

Thus, the price paid or received is integrated into the consequences of the gamble, producing a mixed gamble. Equations 1, 2, and 3 are termed TGPT. Equations 2 and 3 also appeared for maximal buying and minimal selling prices in Birnbaum and Zimmermann (1998, p. 178). [More general expressions appeared in Luce (2000), where subtraction is replaced by an (inverse) joint receipt model. Luce’s (2000) model is evaluated in Birnbaum, et al. (in press). ]

 In the parameterized version of their model, Schmidt, et al. (2008) also assume that utility can be approximated as follows,

(4) *u*(*x*) = *x*, for *x* ≥ 0

(5) *u*(*x*) = – *(– x)*, for *x* < 0

where ** is a constant, sometimes called the “loss aversion” parameter. It is usually found that  > 1 and that 0 <  < 1, but restrictions on these parameters are not necessary to what follows.

*1.1 Complementary Symmetry*

Birnbaum and Zimmermann (1998) showed that Equations 1-5 imply a property called *complementary symmetry* for binary gambles. The maximal buying price, B (WTP) for gamble (*y, p; x*) plus the minimal selling price, S (WTA) for the complementary gamble (*y*, 1 – *p; x*) should be *x* + *y*.

Assuming s 1-5, with the definition, *T*(*p*) = [*W*–(1 – *p*)] /*W*(*p*)](1/), it follows that:

(6) B = [*y* + *T*(*p*)*x*]/[1 + *T*(*p*)]

(7) S = [*x* + *T*(*p*)*y*]/[1 + *T*(*p*)]

Adding Expressions 6 and 7, we have

(8) S + B = *x* + *y*.

Complementary symmetry (Equation 8) thus follows from TGPT with Equations 4 and 5, with any weighting functions. Birnbaum and Zimmermann (1998) proved Equations 1-5 imply complementary symmetry for all gambles of the form (*y*, ½, *x*); and Birnbaum, et al. (in press) deduced Equations (6), (7), and (8) for all (*y*, *p*; *x*). Michal Lewandowski (personal communication, April 23, 2016) reports that he has proved that TGPT (Equations 1, 2, and 3) implies complementary symmetry for any *u*(*x*) function on gains and losses.

*1.2 First Order Stochastic Dominance in Judgments*

Birnbaum (1997) devised a recipe that was tested in choice by Birnbaum and Navarrete (1998) who found that about 70% of undergraduates chose *G* = ($96, .85; $90, .05; $12, .10) over *F* = ($96, ,90; $14, .05; $12, .05), even though *F* dominates *G*. Birnbaum (2005) constructed this recipe to compare descriptive models that satisfy dominance against certain models that violate dominance. Because CPT must satisfy stochastic dominance, evidence that people systematically violate dominance shows that CPT is not an accurate descriptive model of risky decision making.

Stochastic dominance also follows in TGPT: judgments of buying or selling prices should also satisfy first order stochastic dominance in this recipe; that is, the WTP (WTA) of *F* should exceed WTP (WTA) of *G* (see Birnbaum, et al., in press).

*1.3 Violations of Restricted Branch Independence*

 Consider three branch gambles with a fixed probability distribution. Let (*x, y, z*) represent a prospect to win *x* with probability *p*; *y* with probability *q*, and otherwise win *z*. Let B(*x, y, z*) and S(*x, y, z*) represent the judged value of buying and selling prices. Restricted branch independence can be expressed for such three branch gambles as follows:

(9a) S(*x, y, z*) > S(*x', y', z*) if and only if S(*x, y, z'*) > S(*x', y', z'*)

(9b) B(*x, y, z*) > B(*x', y', z*) if and only if B(*x, y, z'*) > B(*x', y', z'*)

This property can be violated by TGPT when the weighting functions are not linear. The manner of violation, however, depends on the weighting function. According to third-generation prospect theory, it should be possible to predict the types of violations of this property in judgments from the shape of the weighting function estimated from choices (Birnbaum, 2008; Birnbaum & Zimmermann, 1998).

 For example, suppose *z'* > *y'* > *y* > *x* > *x' > z* > 0. There are two types of violations for maximum buying prices, Type 1: B(*x, y, z*) > B(*x', y', z*) and B(*x, y, z'*) < B(*x', y', z'*), or Type 2: B(*x, y, z*) < B(*x', y', z*) and B(*x, y, z'*) > B(*x', y', z'*). The same types of violations can describe minimum selling prices. It can be shown that if the decumulative weighting functions are inverse-S in shape (as assumed by Tversky and Kahneman, 1992, Schmidt, et al., 2008, and others), that if a violation is observed (apart from random error), it should only be of Type 2 for either buying or selling prices. For example, the parameterized model of Schmidt, et al. (2008) implies that B($39, $45, $2) < B($12, $96, $2) and B($39, $48, $148) > B($12, $96; $148). An algebraic analysis of restricted branch independence and the shape of the weighting function is presented in Birnbaum and McIntosh (1996) and an algebraic derivation and graphical interpretation is given in Birnbaum (2008, p. 484-487).

The weighting function of CPT required to reproduce standard findings in the literature must have an inverse-S form (Tversky & Kahneman, 1992; Birnbaum, 2008; Schmidt, et al., 2008; Wakker, 2011). Because TGPT requires the same weighting function for both choice and for judgments of value, it predicts the Type 2 pattern of violation of restricted branch independence in choice tasks, and in WTP and WTA judgments.

**2. Empirical Judged Prices (WTP and WTA) violate TGPT**

*2.1 Complementary Symmetry*

 According to TGPT, the sum of buying and selling price of complementary binary gambles should equal the sum of the outcomes. This sum should be independent of other factors, such as the range, |*x* – *y*|, holding *x* + *y* constant. Note that this conclusion follows for any weighting functions, *W* and *W*–.

 At the time of Birnbaum and Sutton (1992), neither third generation prospect theory nor the property of complementary symmetry had yet been developed. However, that experiment allows one to test the property. Figure 1 shows a reanalysis of data from Birnbaum and Sutton (1992) to test complementary symmetry. The figure shows the sum of median judgments of buying and selling prices of gambles of the form (*x*, .5; *y*); that is S + B. These are plotted as a function of |*x* – *y*| with a separate curve for each value of T = *x* + *y*. According to TGPT, the curves should be horizontal with a constant value of *x* + *y*.



Figure 1. Sum of median WTP and WTA (buying plus selling prices) for gambles of the form (x, ½; y) as a function of |x – y| with a separate curve for each level of T = *x* + *y*. According to complementary symmetry, all curves should be horizontal. Instead, all curves decrease as a function of range. (Data from Birnbaum and Sutton, 1992).

Instead, Figure 1 shows that S + B decreases systematically with the range. For example, the median buying and selling prices of ($48, .5; $60) are $50 and $54, respectively, for a total of $104. However, the median buying and selling prices of ($12, .5; $96) are $25 and $50, respectively, for a total of only $75. TGPT implies that both totals should have been $108. For all 28 gambles with positive outcomes studied by Birnbaum and Sutton (1992), median B + S = *x* + *y*, in every case where *x* = *y* and median B + S < *x* + *y* in every other case. Every curve decreases as a function of range (|*x – y*|), so these data systematically violate complementary symmetry, contrary to TGPT. Other violations of complementary symmetry are reported in Birnbaum, et al. (in press), who showed that holding *x* + *y* and |x – y| constant, S + B for complementary gambles varies systematically as a function of *p*.

*2.2 First Order Stochastic Dominance*

 Birnbaum & Yeary (1998) asked 66 undergraduates to evaluate 166 risky gambles from the viewpoints of both highest buying price and lowest selling price. Interspersed among these trials were 8 trials that provided four tests of first order stochastic dominance in each point of view. These four tests of first order stochastic dominance were predicted to show violations, according to configural weight models. Table 1 shows the median judgments of WTP and WTA for these 8 gambles.

In all eight comparisons in Table 1 (four tests by two viewpoints), the dominated gamble (denoted *G-* in Table 1) received higher median judgments than the dominant gamble (*G+*). Means are similar and show the same violations (Birnbaum, et al., in press). The overall mean judgment was $63.31 for dominated gambles (averaged over the four tests and two viewpoints), compared to a mean of $55.11 for dominant gambles. This difference between dominant and dominated gambles is significant, *F*(1, 65) = 20.56; therefore, one can reject the null hypothesis that the dominance relation has no effect on judgments of value, in favor of the rival hypothesis that people assign *higher* judgments to dominated gambles in these specially constructed pairs of gambles.

TABLE 1. Median Judgments in the Tests of Stochastic Dominance.

Buying Prices (WTP)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Test | *G+* | Median WTP | *G–* | Median WTP |
| 1 | .05 .05 .90$12 $14 $96 | 30.0 | .10 .05 .85$12 $90 $96 | 60.0 |
| 2 | .06 .06 .88 $3 $5 $97 | 22.5 | .12 .04 .84 $3 $92 $97 | 50.0 |
| 3 | .02 .03 .95 $6 $8 $99 | 40.0 | .05 .03 .92 $6 $91 $99 | 54.0 |
| 4 | .01 .01 .98 $4 $7 $97 | 50.0 | .02 .02 .96 $4 $89 $97 | 62.5 |

Selling Prices (WTA)

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Test | *G+* | Median WTA | *G–* | Median WTA |
| 1 | .05 .05 .90$12 $14 $96 | 73.5 | .10 .05 .85$12 $90 $96 | 81.5 |
| 2 | .06 .06 .88 $3 $5 $97 | 68.0 | .12 .04 .84 $3 $92 $97 | 80.0 |
| 3 | .02 .03 .95 $6 $8 $99 | 82.5 | .05 .03 .92 $6 $91 $99 | 83.5 |
| 4 | .01 .01 .98 $4 $7 $97 | 81.0 | .02 .02 .96 $4 $89 $97 | 87.5 |

 Analysis of individuals’ data showed that 51 of 66 participants (77%) assigned higher mean judgments to dominated gambles than to the dominant gambles, averaged over the four tests and two viewpoints. Only 15 judges assigned higher mean judgments to the dominant gambles.

 These results with judgment agree with results from direct choices between the same pairs of gambles; Birnbaum and Navarrete (1998) found that 73, 61, 73, and 73 judges (out of 100) chose the dominated gamble *G*– over the dominant gamble, *G+*, in direct choices of Tests 1 though 4 (of Table 1), respectively. In sum, violations of first order stochastic dominance are observed in buying prices, selling prices, and in direct choices. These are all systematic violations of TGPT. Because first order stochastic dominance follows in CPT and TGPT for any utility and any decumulative weighting functions, it is not possible to salvage these models by choosing other parameters, or choosing other functions for utility or weighting.

*2.3 Violations of Restricted Branch Independence*

 Because TGPT assumes that the same *W*–(*p*) and *W*(*p*) functions apply to judgments as to choices, this theory implies that we should be able to predict violations of restricted branch independence from the shape of the probability weighting functions estimated from choice experiments. It is well-known that to describe standard results of empirical choice studies, including the Allais paradoxes, CPT requires inverse-S probability weighting functions in which intermediate branches receive lower weight than lowest or highest valued branches (Tversky & Kahneman, 1992; Birnbaum, 2008; Wakker, 2011).

 Empirically, the observed type of violation of restricted branch independence in both WTP and WTA judgments is not in agreement with the inverse-S weighting function postulated in TGPT (Birnbaum & Beeghley, 1997; Birnbaum & Zimmermann, 1998; Birnbaum & Veira, 1998). In these studies, it is typically found that the violations are of Type 1 rather than Type 2 (See Section 1.3).

 For example, Table 2 shows mean judgments of 12 of the prospects studied by Birnbaum and Beeghley (1997), who asked 46 participants to judge both WTP and WTA for 166 gambles, each of which had three, equally likely outcomes: (*x, y, z*). The mean (and median) judgments violate restricted branch independence in both viewpoints.

 The predicted judgments in Table 2 are calculated from TGPT using the parameters estimated by Tversky and Kahneman (1992). The observed type of violations of restricted branch independence are opposite of predictions. Note that in the WTA viewpoint, S($2, $45, $51) = $34.1 > S($2, $12, $96) = $28.5 and yet S($45, $51, $148) = $62.0 < S($12, $96, $148) = $75.2. However, using TGPT, predictions have the opposite relations. Thus, the data show the Type 1 violation, whereas TGPT with an inverse-S weighting function implies that if systematic violations are observed, they should be of Type 2.

Similarly, in the WTP viewpoint, B($2, $33, $39) = $19.1 > B($2, $12, $96) = $14.4 and B($12, $96, $148) = $47.8 > B($45, $51, $148) = $39.8, and the predictions are again backwards for TGPT. The problem for TGPT is basically the same as for CPT; the inverse-S shaped weighting function implies the opposite type of violations from what is observed empirically.

Violations of restricted branch independence in choice also show the opposite pattern from that predicted by the inverse-S weighting function needed by that theory. This pattern of violations has been replicated in dozens of empirical studies using different formats for presentation of choices (Birnbaum, 2004, 2008; Birnbaum & Bahra, 2012; Birnbaum & Navarrete, 1998). Therefore, one cannot retain both CPT and the inverse-S decumulative weighting function, if one wants to explain either judgment or choice.

Table 2. Reanalysis of Data from Birnbaum and Beeghley (1997). Predicted WTP and WTA are based on third-generation prospect theory using parameters of Tversky and Kahneman (1992).

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Lottery | WTP | WTA | Pred WTP | Pred WTA | EV |
| ($2, $27, $33) | 15.4 | 23.0 | 6.7 | 27.0 | 20.7 |
| ($2, $33 $39) | 19.1 | 26.6 | 7.7 | 33.0 | 24.7 |
| ($2, $39, $45) | 19.6 | 30.0 | 8.6 | 38.9 | 28.7 |
| ($2, $45, $51) | 21.9 | 34.2 | 9.6 | 44.9 | 32.7 |
| ($2, $51, $57) | 27.7 | 37.1 | 10.5 | 50.9 | 36.7 |
| ($2, $12, $96) | 14.4 | 28.5 | 11.9 | 61.5 | 36.7 |
| ($27, $33, $148) | 35.5 | 51.9 | 38.6 | 102.7 | 69.3 |
| ($33, $39, $148) | 39.8 | 50.2 | 44.1 | 105.0 | 73.3 |
| ($39, $45, $148) | 45.2 | 58.5 | 49.6 | 107.3 | 77.3 |
| ($45, $51, $148) | 49.9 | 62.0 | 55.0 | 109.6 | 81.3 |
| ($51, $57, $148) | 56.5 | 68.5 | 60.5 | 111.8 | 85.3 |
| ($12, $96, $148) | 47.8 | 75.2 | 30.7 | 108.0 | 85.3 |

**3. Configural Weighting Models**

 In social and evaluative judgment tasks, it is often the case that unfavorable or negative information seems to override the effects of positive or favorable information. Birnbaum (1974) considered two theories to explain this effect: either the negative information has more extreme value or it has greater weight. In a series of experiments, Birnbaum (1974) concluded that the lower valued or negative information has greater weight, and represented the combination of information by a rank-affected configural weight averaging model:

(10) V(*x, y*) = (.5)*u*(*x*) + (.5)*u*(*y*) +  |*u*(*x*) – *u*(*y*)|

Where V(*x, y*) is the overall value of a stimulus composed of *x* and *y,* two components of equal plausibility, having subjective values of *u*(*x*) and *u*(*y*); and  is the configural weight parameter (–0.5 <  < 0.5) . Note that the weight of the higher ranked value is .5 + , and the weight of the lower ranked value is .5 – . If  = 0, the model is a simple average (EU for 50-50 gambles), but when  is positive, the higher valued stimulus gets greater weight, and when  is negative, the lower valued stimulus gets greater weight. At the extremes, if  = 0.5, or  = – 0.5, the model becomes a maximum or minimum model. For evaluative judgments, it was found that w < 0, indicating greater weight on lower valued aspects or components.

 Birnbaum and Stegner (1979, Experiment 5) theorized that configural transfers of weight (the value of ) would be affected by the judge’s point of view, which they theorized were due to the incentivized consequences to the judge of over- or under-estimating value. To test this configural weight theory, they asked people to judge the most a buyer should pay, the least a seller should accept, or the “fair” price of used cars, based on evaluations provided by people of varied bias and expertise who had examined the cars. Their theory (1979, p. 60-61) was that buyers, sellers, and independents (judging “fair” value) would have different values of  *u*(*x*) is independent of point of view. This model fit the data well, with the assumption that u(x) is independent of viewpoint. This model correctly predicted the finding that WTP and WTA were not monotonically related to each other (Birnbaum, 1982; Birnbaum & Zimmermann, 1998).

 Birnbaum and Stegner (1979) reported that whereas highest buying prices could be fit with  = – 0.19 (greater weight on lower values), lowest selling prices were fit with S = 0.06 (greater weight on higher values). (“Fair” price judgments were fit with F = – 0.07, intermediate between buyer and seller).

*3.1 Configural Weighting Account of Violations of Complementary Symmetry*

 To reproduce violations of complementary symmetry observed by Birnbaum and Sutton (1992), one can simply add simplified configural weight models of buying and selling prices from Birnbaum and Stegner (1979), using Equation 10, with the assumption that *u*(*x*) = *x*:

(11) S(*x, ½; y*) + B(*y, ½; x*) = *x* + *y* + (0.06 – 0.19)|*x* – *y*|

Because  + S = – 0.13, this model predicts the declines in Figure 1 as a function of |*x* – *y*| for each T = *x + y*. Using estimates from Birnbaum and Stegner (1979) is less than optimal, but shows that the result is expected given the configural weight models fit to previous data. Birnbaum and Sutton (1992) estimated  = –0.27, and S = 0.01. Birnbaum, et al. (1992) reported,  = – 0.23, and S = – 0.03. Thus, configural weights represent the asymmetry between buying and selling prices that is reflected in the violations of complementary symmetry.

*3.2 Violations of Stochastic Dominance in Configural Weight Models*

 Although Birnbaum (1974) had used the terminology that “weights depend on ranks” to describe configural weight models such as Equation 10, the configural weight models do not in general imply stochastic dominance, as implied by the class of models that came to be called “rank dependent utility” models in the 1980s (e.g., Quiggin, 1993). To highlight some of the differences between these classes of models, Birnbaum (1997) derived critical tests to distinguish them. Among these tests proposed was a recipe testing first order stochastic dominance; this recipe was tested empirically in judgment by Birnbaum and Yeary (1998) and in choice by Birnbaum and Navarrete (1998).

 The transfer of attention exchange (TAX) model followed the “revised” configural weight model of Birnbaum and Stegner (1979). For three-branch gambles of the form, *G* = (*x, p; y, q; z, 1 – p – q*), *x > y > z* ≥ 0, the model is a weighted average, as follows:

(12) TAX(*G*) = [*Au*(*x*) + *Bu*(*y*) + *Cu*(*z*)]/(*A + B + C*),

where the weights (for branches with highest, middle, and lowest consequences) for a person who puts greater weight on lower consequences (e.g., typical for a buyer) are:

(13a) *A* = *t*(*p*) – 2*t*(*p*)/4;

(13b) *B* = *t*(*q*) + *t*(*p*)/4 – *t*(*q*)/4;

(13c) *C* = *t*(1 – *p* – *q*) + *t*(*p*)/4 + *t*(*q*)/4.

Where *t*(*p*) is a negatively accelerated function of *p* that maps probability of a branch into attention;  is the configural weight transfer parameter, representing a “tax” or transfer of weight from the branches leading to higher consequences to lower consequences. The transfer of weight is proportional to the weight of the branch losing weight; when weight is transferred from lower to higher branches, as is often the case for sellers, the equations need to be rewritten to show transfers in the other direction. If  = 0, the model reduces to a subjectively weighted average utility model, in which weight is a function of probability; if in addition, *t(p*) = *p*, the model reduces to EU.

 Birnbaum’s (1997) recipe is illustrated as follows: Starting with *G0* = ($96, 0.9; $12, 0.1), split the upper branch and reduce the value on the splinter to create a strictly worse gamble *G–* = ($96, .85; $90, .05; $12, .10); now split the lower branch of *G0* and increase the value of the splinter to create a strictly better gamble, *G+* = ($96; .90; $14, .05; $12, .05). By splitting the upper branch to create *G–*, the sum of the weights of the two upper branches has increased (because *t*(*p*) is negatively accelerated), thus improving the value of the gamble (even though it has been made objectively worse); similarly, splitting the lower branch of a gamble makes it seem worse, so *G+* has a lower value despite being objectively better than *G0*. For example, with *t*(*p*) = *p*0.7, *u*(*x*) = *x*, and  = 1, the TAX model values of *G+* and *G–* are $45.77 and $$63.10, respectively; therefore, TAX predicts a violation of first order stochastic dominance in this case.

*3.3 Violations of Restricted Branch Independence*

 Birnbaum (2008) presented an analysis of violations of restricted branch independence in the TAX model (Equations 13a, 13b, and 13c) and showed that if violations occur, they must be of Type 1. If  = 0 (Equations 13a, 13b, 13c), then there will be no violations of restricted branch independence. However, whether weight is transferred from highest valued branches to lower valued branches, or vice versa, if violations occur, they must be of Type 1 (see Birnbaum, 2008, Figure 11).

 Birnbaum and Beeghley (1997) fit a more general, configural weight model that includes both TAX and CPT as special cases. In this model, A, B, and C in Equation 12 are free to vary and estimated separately for WTP and WTA. The weights of lowest, middle, and highest of three equally likely consequences were estimated to be 0.56, 0.36, and 0.08 in the buyer’s viewpoint (WTP), respectively, and they were 0.27, 0.52, and 0.21 in the seller’s viewpoint (WTA), respectively. In both cases, the middle valued branch does not have the least weight, contrary to the inverse-S weighting function of CPT. As shown in Birnbaum and Beeghley, these weights do an excellent job of fitting the violations of restricted branch independence and the non-monotonic relationship between WTP and WTA (including Table 2), even with the assumption that *u*(*x*) = *x*.

*3.4 Model Fitting: Comparison of Fit*

 The TGPT models of WTP and WTA (Equations 6 and 7) were fit to judgments of 63 binary gambles of the form, (*x, p; y*) by Birnbaum, et al. (in press). There were 9 levels of probability and 7 levels of (x, y), so 9 parameters were estimated for the 9 values of *T*(*p*) in Equations 6 and 7. Estimating *T*(*p*) for each *p* allows complete flexibility to the weighting functions, *W* and *W*– so they need not follow the inverse-S shape or any particular form in this analysis;  and  were also free, so there were 11 free parameters.

Despite the flexibility allowed by so many free parameters, TGPT does not fit the data as well as configural weight models that use fewer parameters. The sum of squared deviations between predicted and obtained judgments (126 predicted values for 63 gambles in WTP and WTA) was 20,242 for TGPT (11 parameters) compared to 1,051 for the TAX model with 6 parameters and 1,097 for a TAX model with 5 free parameters (with *u*(*x*) = *x*). This comparison shows how much better the older, configural weight models perform in predicting judgments.

**4. Models of Loss Aversion and Prospect Theories**

 Tversky and Kahneman (1991) proposed a model of the endowment effect for riskless goods. According to this theory, people consider it a loss to pay for a good when buying, and when selling, consider it a loss to give up the item sold. This model, extended to gambles with CPT, is not the same as TGPT. As shown in Birnbaum and Zimmermann (1998), this model implies that the ratio of selling prices to buying prices (WTA/WTP) should be a constant, which TGPT does not predict. Birnbaum and Stegner (1979) had already shown that this ratio is not a constant for judgments of highest selling price or lowest buying prices of used cars, and in fact, buying and selling prices are not monotonically related to each other.

 Although they did not cite the earlier evidence against their theory nor older theories, Tversky and Kahneman (1991) did note that their model was implausible because it implied that the ratio of the selling price of a $5 bill to the buying price of the same bill should be about 4:1. To avoid this obvious flaw, they postulated an exception for goods held for exchange, like cash or gold. But this “exception” is not a problem for the earlier models, which do not have this implication.

 This exception for cash is not required by the model of Birnbaum and Stegner (1979), who had already shown that the ratio of selling to buying prices varies systematically for different “riskless” entities to be evaluated. The “exception” required by the model of Tversky and Kahneman (1991) is not a problem for the configural weight models, which imply that the ratio of selling to buying price will be 1 for the $5 bill and will show systematically greater ratios as a function of the amount of uncertainty or ambiguity of the goods or prospects in question. For a fuller discussion of this point, see Birnbaum, Coffey, Mellers, & Weiss (1992).

 Plott and Zeiler (2005) criticized the experimental methods used in the isolated “endowment effect” literature, as did Birnbaum and Zimmermann (1998). One of the criticisms is that when one studies only a single object (e.g., a mug), it is not possible to test if the ratio of WTA/WTP is a constant or if these measures are monotonically related, nor test more complex properties such as restricted branch independence.

The literature on the “endowment effect” seems to have been limited not only by weak experimental designs and procedures but also by isolation from highly relevant empirical results and theoretical developments in psychology and other fields, as discussed here. A recent *Annual Review of Economics* article (Ericson & Fuster, 2014), for example, cites none of the articles or evidence stemming from the related work that preceded the “endowment” terminology, work discussing weight versus utility (“loss aversion”) theories of the “endowment” effect, nor Luce’s (2000) approach to the topic based on joint receipts.

*4.1 Prospect Theories Refuted in Studies of Direct Choice*

 Both original prospect theory and CPT have been refuted as descriptive models of choice between risky prospects. Although these models could account for those phenomena that had already been published when these models were constructed, they failed to correctly predict new tests of their implications. Both versions of prospect theory can now be rejected by empirical findings in what Birnbaum (2008) calls the “new paradoxes.” These new paradoxes refute prospect theories in the same way that the Allais paradoxes violated EU; some of them hold for any monotonic utility functions and any decumulative weighting functions.

 For example, the violations of first order stochastic dominance reported here in Table 1 have also been found in direct choice. The violations have been observed not only with the displays used in Table 1, but with more than a dozen different formats for presenting choices and representing probability. They have been found with probability represented via balls of different colors in urns, with tickets with different prize values printed on them, with pie charts representing spinners, with bar charts showing probabilities, with lists of equally likely outcomes, with independent and dependent gambles, with decumulative probabilities, and with different arrangements of juxtaposing branches in the gambles compared. They have been observed in quiet lab studies of hypothetical choice, in Internet studies with a chance of real prizes, and in public settings where real cash prizes were awarded publicly via drawings conducted after the choices, generating obvious excitement and interest in the task (reviewed in Birnbaum, 2008; see also Birnbaum and Bahra, 2012).

Besides the violations of first order stochastic dominance, there are six other “new paradoxes” of choice representing theorems of RSDU (including CPT and EU), violations of which refute all forms of the models. These include the probability-outcome tradeoff with indirect dominance, coalescing, gain-loss separability, lower and upper cumulative independence, and upper tail independence. Despite flexibility of choosing any weighting functions and any utility functions for gains and losses, there is no way to use CPT or EU to account for systematic violations of these critical properties. Birnbaum (2008) reviewed the dozens of empirical studies showing violations of these critical properties.

In addition, there are tests of other behavioral properties of choice that contradict the implications of CPT with its inverse-S weighting function. Violations of restricted branch independence are also observed in choice experiments and show the same type of violation (Type 1) reported here in Table 2. There are now dozens of studies using different display formats, probability representations, and incentives that show the same prevalent type of violation of restricted branch independence in choice studies. Birnbaum (2008) also summarized many of these studies, as well as others that test other behavioral properties that also contradict the weighting function of CPT, such as tests of distribution independence, and the dissection of the Allais paradox (Birnbaum, 2004).

The fact that violations of stochastic dominance and restricted branch independence yield such similar results in both judgment tasks and in direct choices suggests that something common to both tasks is likely involved.

There are some theorists who believe that anomalous findings that refute a model are found (perhaps by chance), which lead to new revised theories to explain them. It is worth emphasizing, to set the historical record straight, that in this case, the configural weight models were developed before the RSDU (and CPT) models, and were used to design the tests of critical properties that refuted those newer models.

*4.2 Cognitive Process Theories of Evaluation*

Johnson and Busemeyer (2005) developed a process model of attention that can be interpreted as a cognitive theory to account for the configural weights in Birnbaum and Beeghley’s (1997) configural weights. Their model achieves a fit comparable to that of Birnbaum and Beeghley. Ashby, Dickert, & Glöckner (2012) examined the time that people spend looking at the possible outcomes when forming judgments of value. They found that in the buyer’s viewpoint, more time was spent looking at the lowest possible prize than spent by those in the seller’s viewpoint at all levels of probability to receive the lower prize. This finding would be consistent with the idea that time spent looking might be an indicator of the attention or weight given to an item.

**5. Conclusions**

 TGPT is not an accurate descriptive model of the endowment effect and of preference reversals because it implies properties of buying prices, selling prices, and choices that are systematically violated by empirical data. These violations are large, robust, and have been replicated in multiple experiments.

Configural weight models (Birnbaum & Stegner, 1979), remain compatible with major properties of empirical data that refute the loss aversion theory of Tversky and Kahneman (1991) and that refute TGPT (Schmidt, et al., 2008).

Birnbaum and Stegner’s (1979) model can explain their finding that the ratio of WTA to WTP is not a constant and that these two judgments are not even monotonically related to each other. It correctly predicted the violations of complementary symmetry in Figure 1, it correctly describes violations of restricted branch independence, and it fits data better than the parameterized model of Schmidt, et al. (2008).

It seems reasonable to ask those who would continue to work with the concept of loss aversion in connection with the endowment effect to show that their theory provides a better fit to the data than earlier models that had been proposed to account for the effect or to develop a new theory using the shape of the utility function (“loss aversion”) to account for the empirical results of experiments testing formal properties such as first order stochastic dominance, restricted branch independence, complementary symmetry, and monotonicity between WTP and WTA. Those working with the concept of loss aversion should respond to the challenge to demonstrate that it provides a better description of data than provided by earlier configural weight models.

Configural weighting models correctly predicted violations of complementary symmetry, which violates TGPT. Configural weight models were used to design the gambles that violate stochastic dominance in WTP, WTA, and choice. Those models correctly predicted violations of restricted branch independence, which are in the opposite direction from what is predicted by TGPT, and configural weighting models provide a better quantitative fit to data. They also account for the fact that the ratio of selling to buying prices is not a constant and that these two kinds of judgments are not monotonically related to each other, contrary to the theory of Tversky and Kahneman (1991).

 In configural weight models, the utility of a gamble is a weighted average of the utilities of the consequences on the branches of the gambles. Those models need only two basic ideas regarding configural weights to account for the phenomena described here, besides the properties of an averaging model.

The first idea is that absolute weight of a branch in a gamble (apart from configural effects) is a nonlinear function of its probability.

The second idea is that weight of a branch is affected by the rank of the consequence on the branch compared to other branch consequences. In the TAX model, weight is transferred among branches in proportion to the probability weight of the branch losing weight. As in Birnbaum and Stegner (1979), weight is transferred from branches leading to higher valued consequences to lower ones in the case of buying prices (WTP) and transferred from lower to higher branches in selling prices. That is, buyers put more weight on the lower valued aspects or possibilities of an object or gamble and sellers place greater weight on higher valued aspects or possible outcomes of an object or gamble.

With these two basic ideas of configural weighting, one can account for results that violate TGPT and CPT. It is not necessary to assume that utilities change or that utilities of losses play any role in describing the phenomena of buying and selling of lotteries on positive consequences.

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