

5. The Vexed Question of Rational Self-interest

Economics as a discipline is dominated by a single theoretical idea: rational self-interest. Much of economics is theoretical, and much of its theory consists of working out what a rationally self-interested individual would do in a given situation. Psychology, by contrast, has no dominant theory, and its dominant research paradigm is to submit theoretical assertions to empirical test.

Unsurprisingly, two such different disciplines have made uncomfortable bedfellows. Furthermore, the empirical bias of psychologists has led them, as soon as they start to look at economic behavior, to question the economic assumption of rational self-interest. This means that a great deal of research in economic psychology has consisted of investigations by psychologists of the empirical accuracy of theories developed by economists. A smaller amount has consisted of the incorporation into economic theory of psychological principles.

Economic rationality is a protean concept; some have argued, indeed, that it is not a theory at all, more of a general language within which specific theories can be expressed. Economists are not necessarily impressed when psychologists find that, in artificial experiments, some percentage of individuals show a percentage variation from the truly rational behavior pattern: they protest that they are interested in predicting the broad trends of behavior in the great mass of people, so as to explain the performance of the economy as a whole—and they argue that many empirical deviations from rationality are second-order effects of little or no macroeconomic consequence.

Recognizing the various difficulties associated with a frontal assault on rationality assumptions, modern economic psychologists mostly do not test rationality assumptions directly, but rather try to build empirically valid models of the causation and consequences of economic behaviors. Popular causal variables include personality differences, attitudes, socialization experiences, and psychological disorders. All these variables can potentially cause different people, exposed to apparently identical economic situations, to react in different ways. They thus allow psychologists to account for some of the individual variation in behavior that remains when the obvious economic variables have been taken into account.

Cumulatively, of course, such studies can still constitute a powerful attack on the economic theory of rational self-interest. There are many ways of acting rationally in any given situation, depending on your knowledge, abilities, values, and goals. If most of the variation between people's economic behavior actually depends on which rational behavior people choose, and if this has to be predicted from psychological properties of the person, the role of economic psychology has to increase, while the role of rational theory as such has to decrease. How far that process

will go remains uncertain; what unites economic psychologists is a belief, based on the fruitfulness of research so far, that it has to go further than it yet has.

See also: Bayesian Theory: History of Applications; Bounded and Costly Rationality; Decision Research: Behavioral; Heuristics for Decision and Choice; Psychology and Economics; Risk: Empirical Studies on Decision and Choice; Utility and Subjective Probability: Contemporary Theories; Utility and Subjective Probability: Empirical Studies

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Decision and Choice: Paradoxes of Choice

Choice paradoxes are cases where people make decisions that systematically violate the implications of a theory that is considered (at the time of the 'paradox') a viable representation of human decision-making. In the days of the classic paradoxes, a rational person's choices were thought to be normative and so paradoxes of choice also influenced theoreticians' ideas of what is rational. Results considered paradoxical from the viewpoint of one theory are explained by newer theories proposed to provide more accurate representations of decision-making. Modern paradoxes are violations of clearly stated premise or implications of a descriptive model.

1. The Classic Paradoxes

Before 1738, it was considered that a rational person should choose between gambles according to their expected values. The expected value (EV) of a gamble with n mutually exclusive and exhaustive possible outcomes can be written:

$$EV = \sum_{i=1}^n p_i x_i \quad (1)$$

where p_i and x_i are the probability and prize for outcome i .

1.1 St Petersburg Paradox Refutes EV

The St Petersburg paradox presents a case where scholars insisted that they would and should prefer a small amount of cash to a gamble with much higher EV . Consider the following gamble: a fair coin will be tossed and if the outcome is heads, the prize is \$2 and the game ends; however, if it is tails, the coin will be tossed again. If the second toss is then heads, the prize is \$4; if tails, the coin is tossed again, and the prize for heads doubles for each successive tails that occurs before heads. When heads occurs, the prize is awarded and the game ends. The EV of this St Petersburg gamble is infinite,

$$EV = \sum_{i=1}^{\infty} p_i x_i = \frac{1}{2} 2 + \frac{1}{4} 4 + \frac{1}{8} 8 + \dots = \infty$$

Therefore, if one prefers higher EV , then one should prefer this gamble to any finite amount of cash. Instead, most people say that they would prefer a small sum (e.g., \$15) rather than play this gamble once.

1.2 EU Explains St Petersburg Paradox

This paradox, or contradiction between EV and human judgment, was explained by Bernoulli (1738), who showed that if utility is a nonlinear function of wealth, then the expected utility (EU) of the gamble might indeed be less than the utility of a finite sum of cash. In EU theory:

$$EU = \sum_{i=1}^n p_i u(x_i) \quad (2)$$

Bernoulli went on to show that EU theory could explain why a pauper might be willing to sell a gamble to a wealthy person for less than its EV and why both would consider the exchange rational. He also explained the purchase and sales of insurance.

When von Neumann and Morgenstern (1947) developed axiomatic foundations of EU and when

Savage (1954) developed subjective expected utility theory (SEU), generalizing EU to cases of uncertainty where objective probabilities are unspecified, violations of EV no longer seemed paradoxical. Risk-averse behavior (preferring sure cash over a gamble with the same or higher EV) and risk-seeking behavior (preferring the gamble over the EV in cash) could be explained by the shapes of the utility functions for different cases. It was not long, however, before new paradoxes were discovered that confounded EU and SEU .

1.3 The Paradoxes of Allais Violate EU

Allais suggested two-choice problems in which no utility function could be constructed that would explain choices (Allais and Hagen, 1979). The constant consequence paradox is illustrated with the following two choices:

- A : \$1 million for sure
 B : 0.01 probability to win \$0
 0.89 probability to win \$1 million
 0.10 probability to win \$5 million
 A' : 0.89 probability to win \$0
 0.11 probability to win \$1 million
 B' : 0.90 probability to win \$0
 0.10 probability to win \$5 million

Choices A versus B and A' versus B' differ only by changing the common consequence on a .89 chance to win \$1 million in both sides of the first choice to \$0 in the second choice. Many people preferred A to B and also preferred B' to A' , contrary to EU .

Allais also proposed a constant ratio paradox, which can be illustrated as follows:

- C : \$3,000 for sure
 D : 0.20 probability to win \$0
 0.80 probability to win \$4,000
 C' : 0.75 probability to win \$0
 0.25 probability to win \$3,000
 D' : 0.80 probability to win \$0
 0.20 probability to win \$4,000

Note that the probability to win is one-fourth in C' versus D' , compared to C versus D . Many people persisted in choosing C over D and D' over C' , contradicting EU theory.

1.4 SEU and the Ellsberg Paradoxes

Consider an option with n mutually exclusive and exhaustive events, where each event, E_i , leads to consequence C_i with subjective probability $S(E_i)$. Savage's (1954) SEU replaces probability, p_i , in Equation 2 with subjective probability, $S(E_i)$.

Although SEU has two subjective functions and may therefore seem hard to test, Ellsberg (1961)

Table 1

Choices illustrating Ellsberg paradox

Option	Payoffs for drawing a ball of each color		
	<i>Red</i>	<i>Blue</i>	<i>Green</i>
<i>F</i>	\$100	\$0	\$0
<i>G</i>	\$0	\$100	\$0
<i>F'</i>	\$100	\$0	\$100
<i>G'</i>	\$0	\$100	\$100

devised paradoxes contradicting SEU theory. Ellsberg's paradox can be illustrated as follows. Suppose there is an urn with 90 balls, 30 of which are *Red*, and 60 of which are either *Blue* or *Green*, in unknown proportion. Now consider Table 1.

A person who prefers *F* to *G* should also prefer *F'* to *G'*, since the only difference is the (constant) consequence for a Green ball, which is the same within each choice. If a person prefers *F* to *G*, the theory implies that $S(\textit{Red}) > S(\textit{Blue})$, and if that person prefers *G'* to *F'*, then $S(\textit{Red}) < S(\textit{Blue})$, a contradiction. Indeed, many people exhibited this paradoxical choice pattern even when confronted with this argument. One interpretation is that people are averse to ambiguity, as well as to risk. Others suggested that the decider might distrust that the urns are identical in both choices.

1.5 Paradoxical Risk Attitudes

In *EU* theory, the shape of the utility function determines risk attitudes. For example, for $x > 0$, if $u(x) = x^b$, then the person should be risk-averse if $b < 1$, and risk-seeking if $b > 1$. However, many people are both risk-seeking, when p is small, and risk-averse, when p is moderate to large. Furthermore, many people show risk-aversion for small probabilities of heavy losses (they buy insurance) and they accept risks to avoid certain or highly probable losses.

Whereas Allais considered paradoxical choices 'rational', and theory to be wrong, Savage considered paradoxical choices to be human 'errors' that should be corrected by theory. Many psychologists considered the contradiction between theory and behavior to mean that descriptive theory need not be rational. In this purely descriptive approach, a choice paradox is merely a clear contradiction between theory and human behavior.

Paradoxical risk attitudes and the Allais paradoxes can be described by a theory in which decision weights are a function of probabilities (Edwards 1954, Kahneman and Tversky 1979). Prospect theory (Kahneman and Tversky 1979) described many of the empirical phenomena known by the 1970s. However, this theory was restricted to gambles with no more than two non-zero payoffs and it included a number of seemingly *ad hoc* editing rules to avoid implications

that were considered both irrational and empirically wrong.

2. Modern Theories and Paradoxes

During the 1980s, a number of investigators converged on an approach that used a weighting function of probability but did not have the problems of prospect theory (See reviews by Quiggin 1993, Luce 2000). This class of models includes rank-dependent expected utility (Quiggin 1993), rank- and sign-dependent utility theory (Luce 2000), and cumulative prospect theory (Tversky and Kahneman 1992, Wakker and Tversky 1993, Wu and Gonzalez 1998), among others.

2.1 Rank-dependent Expected Utility

For gambles with nonnegative consequences, the rank-dependent expected utility (RDEU) of a gamble can be written as follows:

$$RDEU(G) = \sum_{i=1}^n \left[W \left(\sum_{j=i}^n p_j \right) - W \left(\sum_{j=i+1}^n p_j \right) \right] u(x_i) \quad (3)$$

The consequences are ranked such that $0 \leq x_1 < x_2 < x_3 < \dots < x_n$; and W is a strictly monotonic weighting function with $W(0) = 0$ and $W(1) = 1$. This representation (and its extensions to the cases of negative outcomes and uncertain events) can handle the classic paradoxes of Allais and Ellsberg.

However, new paradoxes were soon created to test if rank-dependent models are descriptive of choices that people make. The new paradoxes can be analyzed as the result of combinations of simpler properties.

2.2 Transitivity, Monotonicity, Coalescing, and Branch Independence

Transitivity holds if for all gambles, $A \succ B$ and $B \succ C$ implies $A \succ C$, where $A \succ B$ denotes A is preferred to B .

Monotonicity assumes that, if one consequence of a gamble is improved, holding everything else in the gamble constant, the gamble's utility is improved.

Coalescing holds that, if two (probability-consequence) branches of a gamble yield the same consequence, then such branches can be combined by adding their probabilities without affecting the gamble's utility. For example, $(\$0, 0.8; \$100, 0.1; \$100, 0.1)$ ought to have the same utility as $(\$0, 0.8; \$100, 0.2)$.

Branch independence is the assumption that if two gambles have a common consequence for a given state of the world with known probability, then the value of

the consequence on that common branch should have no effect on the preference order induced by other components of the gambles. Branch independence is weaker than Savage's (1954) 'sure thing' axiom because it does not presume coalescing. It was assumed as an editing rule by Kahneman and Tversky (1979).

Let $G = (x, p; y, q; z, r)$ represent the three-outcome gamble to win x with probability p , y with probability q , and z otherwise ($r = 1 - p - q$), where $0 < x < y < z$. For such gambles, restricted branch independence implies,

$$\begin{aligned} (x, p; y, q; z, r) &\succ (x', p; y', q; z, r) \\ &\text{if and only if} \\ (x, p; y, q; z', r) &\succ (x', p; y', q; z', r) \end{aligned} \quad (4)$$

where the branches are distinct, the probabilities are non-zero and they sum to 1. The term 'restricted' refers to the constraint that the probabilities and number of outcomes are fixed in all of the gambles. Although branch independence is implied by *EV*, *EU*, *SEU*, and certain other theories, it can be violated by RDEU (equation 3) if the common consequence (z or z') changes rank. However, if the common consequence maintains the same rank (i.e., the same cumulative probability), the case is termed comonotonic branch independence, which is implied by Eqn. 3.

One can view the constant consequence paradox of Allais as a violation of (non-comonotonic) restricted branch independence, coalescing, and transitivity. Thus, a theory that violates either branch independence (such as RDEU) or coalescing can explain this Allais paradox.

2.3 Paradoxical Violations of Branch Independence

RDEU (Eqn. (3)) accounts for the observed pattern of risk-seeking for small probabilities to win big prizes and risk-aversion for medium to large probabilities to win modest positive consequences (Tversky and Kahneman 1992). It also accounts for Allais paradoxes if the cumulative weighting function, W in Eqn. (4), has an inverse-S shape (Wu and Gonzalez 1998) in which a probability change near zero or 1 has a greater effect than the same change near 1/2.

However, Birnbaum and McIntosh (1996) reported a pattern of violation of branch independence opposite to the prediction of this inverse-S weighting function. This pattern can be illustrated as follows:

- S*: 0.80 probability to win \$2
 0.10 probability to win \$40
 0.10 probability to win \$44
- R*: 0.80 probability to win \$2
 0.10 probability to win \$10
 0.10 probability to win \$96

Choices S' and R' differ only in the consequence on the common branch of probability 0.8.

- S' : 0.10 probability to win \$40
 0.10 probability to win \$44
 0.80 probability to win \$110
- R' : 0.10 probability to win \$12
 0.10 probability to win \$90
 0.80 probability to win \$110

According to the inverse-S weighting function, RDEU implies that violations of branch independence should follow a pattern of $R \succ S$ and $S' \succ R'$. However, the opposite pattern occurs significantly more often than this pattern predicted by the weighting function of RDEU (see review in Birnbaum 1999). Although violations are compatible with Eqn. 3, the observed pattern of violations is the opposite of what was assumed in order to fit Allais paradoxes.

2.4 Configurally Weighted Utility Models

Birnbaum (1997) reviewed configural weight models that he and his colleagues had used in previous research. These models satisfy neither restricted branch independence nor coalescing. They reconcile data that seemed to imply the inverse-S and data that seemed to imply the opposite. Birnbaum's models and the RDEU class of models (Eqn. (3)) are related by being different special cases of the following configural weight model:

$$CWU(G) = \sum_{i=1}^n w(x_i, G)u(x_i) \quad (5)$$

where $w(x_i, G)$ is the configural weight of consequence x_i in gamble G , and $u(x)$ is the utility function. In the transfer of attention exchange (TAX) model, lower-valued consequences 'tax' weight from higher valued ones. A simple version of this model, with $u(x) = x$ for $0 < x < \$150$ has given a good approximation to empirical choices by undergraduates. In this model, the weight of a distinct branch (probability-consequence pair) is proportional to p^7 , which is modified by the configural tax. In two-outcome gambles, the lower branch takes 1/3 of the weight of the higher valued branch; in three-outcome gambles, any lower branch takes 1/4 of the weight of any higher-valued branch. Birnbaum (1997) proposed tests between this model and RDEU.

2.5 Paradoxical Violations of Stochastic Dominance and Event-splitting

Birnbaum (1997) proposed the following choice:

- I*: 0.05 probability to win \$12
 0.05 probability to win \$14
 0.90 probability to win \$96

J: 0.10 probability to win \$12
 0.05 probability to win \$90
 0.85 probability to win \$96

Gamble *I* stochastically dominates *J* because the probability of getting a higher prize than *t* given gamble *I* exceeds or equals that of gamble *J* for all *t*. *EV*, *EU*, and *RDEU* imply that people should choose *I* over *J*. The *TAX* model, fit to previous data, implies people will choose *J*.

Birnbaum and Navarrete (1998) tested this prediction and found that about 70 percent of college students violated stochastic dominance on this and similar choices, in violation of *RDEU*. *RDEU* implies coalescing, monotonicity, and transitivity, so it satisfies stochastic dominance. Configural weight models violate coalescing, and violate stochastic dominance for these cases.

According to the *TAX* model, it should be possible to eliminate the violations of stochastic dominance by event-splitting, as in the following choice:

IS: 0.05 probability to win \$12
 0.05 probability to win \$14
 0.05 probability to win \$96
 0.85 probability to win \$96

JS: 0.05 probability to win \$12
 0.05 probability to win \$12
 0.05 probability to win \$90
 0.85 probability to win \$96

Note that *IS* and *JS* are simply a split versions of *I* and *J*. According to coalescing (and thus *RDEU*), choices between *I* and *J* and between *IS* and *JS* are the same. More than half of the undergraduates tested chose *J* over *I* and *IS* over *JS*, contrary to coalescing (Birnbaum, 1999).

Wu (1994) found violations of tail independence, also called ordinal independence, which can be derived as a combination of coalescing, transitivity, and comonotonic branch independence.

2.6 Violations of Lower and Upper Cumulative Independence

To characterize the paradoxical relationship between evidence that appeared to imply an inverse-S weighting function and evidence that appeared to contradict it, Birnbaum (1997) deduced two new paradoxes, which he called lower and upper cumulative independence. Consider the following choice:

S: 0.80 probability to win \$2
 0.10 probability to win \$40
 0.10 probability to win \$44

R: 0.80 probability to win \$2
 0.10 probability to win \$10
 0.10 probability to win \$98

Now increase the common prize of \$2 to \$10 on the 0.8 branch of both gambles and coalesce it to the \$10

branch in *R*. Now, increase the consequence of \$40 to \$44 and coalesce it with the \$44 branch on the left. If a person preferred *S* to *R*, they should definitely prefer *S'* to *R'*:

S': 0.80 probability to win \$10
 0.20 probability to win \$44

R': 0.90 probability to win \$10
 0.10 probability to win \$98

However, more people switch preference from *S* to *R'* than from *R* to *S'*, contrary to monotonicity!

The following choice illustrates upper cumulative independence:

S': 0.10 probability to win \$40
 0.10 probability to win \$44
 0.80 probability to win \$110

R': 0.10 probability to win \$10
 0.10 probability to win \$98
 0.80 probability to win \$110

If a person prefers *R'* to *S'*, then they should prefer *R''* to *S''*:

S'': 0.20 probability to win \$40
 0.80 probability to win \$110

R'': 0.10 probability to win \$10
 0.90 probability to win \$98

Note that the common branch has been reduced in value to \$98 in both gambles, but the consequence of \$44 has been reduced to \$40 in gamble *S''*, which should make that gamble relatively worse. More people switched from *R'* to *S''* than the opposite (Birnbaum 1999, Birnbaum and Navarrete 1998).

These modern paradoxes, which violate *RDEU*, were predicted by configural weight models. Undoubtedly, new paradoxes will be developed to confound new theories. Each new finding represents a phenomenon that must be explained by newer theories. Although some are pessimistic that any transitive theory will be able to explain all of the paradoxes, others continue the search for a theory that can explain the widest domain of behavior.

See also: Bayesian Theory: History of Applications; Decision and Choice: Random Utility Models of Choice and Response Time; Decision Biases, Cognitive Psychology of; Decision Making, Psychology of; Decision Research: Behavioral; Decision Theory: Bayesian; Decision Theory: Classical; Game Theory; Probability and Chance: Philosophical Aspects; Rational Choice Explanation: Philosophical Aspects; Subjective Probability Judgments; Utility and Subjective Probability: Empirical Studies

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Decision and Choice: Random Utility Models of Choice and Response Time

Introduction

The task of choosing a single ‘best’ option from some available, potentially infinite set of options R has received considerable study in psychology and economics. Random utility models of choice account for the stochastic variability underlying these choices (i.e., the same choice is not necessarily made on repeated presentations of the same set of options) by assuming that there exist a random variable $U(x)$ for each option x and a joint probability measure for these random

variables such that the probability of choosing a particular option x from the set of available options is equal to the probability that $U(x)$ takes on a value greater or equal to the values of all other random variables (e.g., see *Luce’s Choice Axiom; Utility and Subjective Probability: Contemporary Theories*). The basic choice paradigm is extended here by considering, in addition to the option chosen, the point in time that a choice is made from the set of available options. The random utility model can be tailored to cover this situation by replacing the utility variable $U(x)$ by $V(x) = \phi[U(x)]$ (ϕ some monotonically decreasing transformation), where $V(x)$ can be interpreted as decision time for choosing option x , and by replacing the maximum utility rule by a minimum decision time rule: the option chosen is the one which happens to be associated with the minimum choice (or decision) time with respect to all options in the available set. This model will be referred to as ‘horse race’ random utility model.

A number of problems arise in the study of ‘horse race’ random utility models for choice and response time that have found only partial solutions: (a) What conditions on the observable choice probabilities and decision times are necessary and sufficient for a random utility representation? (b) What are the consequences of assuming stochastic independence between the time of choice and the identity of the option chosen, and which assumptions on the joint distribution function do imply this independence? (c) What are possible generalizations to other choice paradigms? The presentation here is partly based on Marley and Colonius 1992 and Marley 1992, 1989. Some related results (not explicitly referred to in the following) can be found in Bundesen 1993, Robertson and Strauss 1981, and Vorberg 1991.

1. ‘Horse Race’ Random Utility Models

Let $R = \{x, y, \dots\}$ be a finite set of potential choice options (for an extension of the theory to infinite choice sets, see Resnick and Roy 1992); the subset X of R containing at least two elements is the currently available choice set. $T(X)$ is a random variable denoting the time at which a choice is made; and $C(X)$ is a random variable denoting the element chosen from X . For $t \geq 0$

$$P_x(x; t) = Pr[C(X) = x \cap T(X) > t]$$

is the probability that options x is chosen from X after time t , and $\{P_x(x; t): x \in X \subseteq R\}$, or (R, P) for short, is called joint structure of choice probabilities and response times. (R, P) is called complete if it is defined for all subsets of X of R with $|X| \geq 2$.

A complete joint structure of choice probabilities and response times (R, P) is said to satisfy a ‘horse race’ random utility model if for any $X \subseteq R$ with

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