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The impact of middle outcomes on lottery valuations☆

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ABSTRACT

This paper presents the results of two experiments that violate implications of expected utility (EU) and cumulative prospect theories (CPT). First, some lotteries with three equally likely branches are valued more than strictly better or objectively equivalent binary lotteries, while others are valued less than strictly worse or objectively equivalent binary lotteries. Second, experimental data provide evidence that lottery valuations strongly depend on the value of the middle monetary outcome(s), whereas CPT as fit to binary lotteries implies that middle outcomes are given lower weights relative to extreme ones. This leads to self-contradiction when CPT is used to fit the data: the probability weighting function takes an inverse S-shape when estimated using binary lotteries, and an S-shape when estimated using lotteries with three or four branches. Both effects are replicated with four-branch lotteries, with different highest outcome values, and with subjects from both Poland and California. It is argued that the violations of coalescing and stochastic dominance observed in the experiments cannot be explained by any rank-dependent weighted utility model, including CPT, but can be described by other rank-affected weighted utility models.

1. Introduction

Imagine you have to determine the certainty equivalents (CE) of two lotteries: (0, ⅓; 300, ⅔) and (0, ⅓; 300, ⅓; 300, ⅓). The two lotteries are objectively equivalent: both offer a ⅔ chance of winning 300 and a ⅓ chance of winning 0; the only difference is that the probability of winning the prize of 300 is split into two equal parts in the latter case. Normatively, the two lotteries should have the same CE. However, as shown in this paper, the latter is valued about 15% more than the former when presented separately in an experiment. Such effects are called "event-splitting" effects (e.g., Starmer and Sugden, 1993).

Splitting effects violate the principle of coalescing, which assumes that splitting or coalescing lottery branches that offer the same monetary outcomes should not make any difference to the lottery valuation. Coalescing is implied by expected utility (EU), and any rank-dependent utility (including cumulative prospect theory, CPT, Tversky and Kahneman, 1992). Violations of coalescing also contradict the original prospect theory (OPT, Kahneman and Tversky, 1979), which postulated that people would use the editing rule of combination to convert a three-branch gamble such as (0, ⅓; 300, ⅓; 300, ⅓) to its equivalent, two-branch form (0, ⅓; 300, ⅔), prior to evaluating it. Using OPT without the editing phase, i.e. the model introduced by Edwards (1962) and then reconsidered by Humphrey (2001), splitting can only increase the lottery valuation (in the case of nonnegative monetary outcomes). However, we also find that splitting the lower branch can make a gamble worse: lottery (0, ⅓; 0, ⅓; 300, ⅓) is valued about 15% less than the lottery (0, ⅔; 300, ⅓). That the lottery valuation may either increase or decrease depending on which branch is split was predicted by the class of configural weight models that had appeared in psychology even before OPT (Birnbaum, 1974; Birnbaum and Stegner, 1979). Clearly, neither EU nor CPT can predict this pattern since they do not violate coalescing at all.

We also find violations of stochastic dominance. For example, the three-branch lottery (0, ⅓; 225, ⅓; 300, ⅓) is stochastically dominated by the two-branch lottery (0, ⅓; 300, ⅔). Despite this, the dominated gamble is given a higher CE value. On the other hand, the three-branch lottery (0, ⅓; 30, ⅓; 300, ⅓) dominates the two-branch lottery (0, ⅔; 300, ⅓). However, the dominant gamble is given a lower CE value.

Testing stochastic dominance had been suggested by Birnbaum (1997) as a way of comparing the class of rank- and sign-dependent utility models (including CPT) that satisfy stochastic dominance, against an older class of configural weight models, which predicts stochastic dominance violations in specially constructed choices. Birnbaum and Navarrete (1998) found that about 70% of college

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undergraduates preferred (\$12, 0.10; \$90, 0.05; \$96, 0.85) over (\$12, 0.05; \$14, 0.05; \$96, 0.90), despite the former being stochastically dominated by the latter.[1] More than 40 studies have since documented stochastic dominance violations in choice testing problems. These studies used different subjects, choice problems, and methods to display the choices and methods used to describe and display probabilities (Birnbaum, 2007, 2008; Birnbaum and Bahra, 2012). Luce (2010) noted that, despite such strong evidence of the core assumptions of CPT being violated, "aficionados of CPT simply dismiss or ignore Birnbaum's findings" (page 21).

Birnbaum and his colleagues have also observed patterns of behavior that refute the inverse-S probability weighting function assumed in CPT. This shape is required to account for the classic Allais paradoxes and standard findings, such as that many people are risk seeking for small probabilities of winning a positive prize, and risk averse for large probabilities. The inverse-S shaped probability weighting function assumed in CPT, when applied to lotteries involving more than two equally likely outcomes, implies that middle ranked outcomes (of a given probability) are given lower weights than extreme ones (given the same probability). Birnbaum and Beeghley (1997) studied lotteries with three equally likely branches (*x, y, z*). The authors observed that lottery (z, \$39, \$45) is valued more than (z, \$12, \$96) when z = \$4, but when the "common" branch of z = \$4 is replaced by z = \$136, the lottery (\$39, \$45, \$136) is valued less than (\$12, \$96, \$136). This result cannot be explained by middle branches being given *lower* weights than both extreme branches, but could be reconciled with CPT with the opposite pattern, that is an S-shaped probability weighting function. Birnbaum and Veira (1998) found similar results for four-branch gambles. Similar results have also been observed in many studies of choice (Birnbaum and McIntosh, 1996; Birnbaum and Navarrete, 1998; Birnbaum and Bahra, 2012).

This paper reports new examples of violation of coalescing and stochastic dominance, as well as evidence that middle outcomes apparently being overweighted (relative to extreme ones) in judgments of certainty equivalents. The idea behind the experiment (Section 2) is loosely based on Birnbaum's method of manipulating one or two outcome values while holding the others constant. There are, however, differences in experimental method and design. First, in the present experiments, only the middle outcome(s) are manipulated. This ensures that the range of lottery outcomes remains unchanged, which helps avoid possible range effects (Kontek and Lewandowski, 2018) that could have an impact on valuations. Second, the experiments concurrently involve two-, three-, and four-outcome lotteries, allowing direct comparisons of valuations of lotteries with different numbers of outcomes. We use CPT as an analytic device to show that CPT and the data lead to self-contradiction: the probability weighting function obtained using binary lotteries takes an inverse S-shape, as postulated by CPT; that estimated using multi-outcome lotteries takes an S-shape, which assigns higher weights to middle outcomes relative to extreme ones. Third, the violations and patterns previously reported in dedicated studies are now captured in a single experiment using judged certainty equivalents instead of choices between lotteries.

Section 3 describes Experiment 1, which was conducted with subjects from Poland, and Section 4 describes Experiment 2, conducted with subjects from California and Poland. The results are discussed in Section 5. It is argued that the data are not consistent with CPT or EU models, which cannot describe violations of coalescing or stochastic dominance, and whose estimated weights lead to self-contradiction. Possible ways of describing the data are presented in the discussion. Appendixes 1 and 2 include the aggregated CE values obtained in the experiments. Appendixes 3 and 4 detail the instructions used in the two experiments.

2. Experiment – the idea

Consider a lottery with three equally likely branches, $L_3 = (x_{min},\ 1/3;\ x,\ 1/3;\ x_{max},\ 1/3)$, where $x_{min} \le x \le x_{max}$. Note that when $x = x_{min}$, the total probability of winning x_{min} is 2/3 and the probability of winning x_{max} is 1/3. The lottery L_3 is therefore objectively equivalent to the binary lottery $L_{2L} = (x_{min},\ 2/3;\ x_{max},\ 1/3)$ in this case. When $x = x_{max}$, the total probability of winning x_{max} is 2/3, and the probability of winning x_{min} is 1/3; in this case the lottery L_3 is objectively equivalent to the binary lottery $L_{2H} = (x_{min},\ 1/3;\ x_{max},\ 2/3)$. In the experiment, the outcome x in L_3varies in the range (x_{min}, x_{max}) and the lottery CE is determined. According to CPT (and other models assuming coalescing), the certainty equivalent of L_3 should assume a value between the CEs of L_{2L} and L_{2H} for each x in the range (x_{min}, x_{max}).

The same idea is applied to a four-branch lottery $L_4 = (x_{min},\ 1/4;\ x_2,\ 1/4;\ x_3,\ 1/4;\ x_{max},\ 1/4)$, where $x_{min} \le x_2 \le x_3 \le x_{max}$. When $x_2 = x_3 = x_{min}$, the probability of winning x_{min} is 3/4 and the probability of winning x_{max} is 1/4. The lottery L_4 is objectively equivalent to the binary lottery $L'_{2L} = (x_{min},\ 3/4;\ x_{max},\ 1/4)$ in this case. When $x_2 = x_3 = x_{max}$, the total probability of winning x_{max} is 3/4, and the probability of winning x_{min} is 1/4, so that L_4 is objectively equivalent to $L'_{2H} = (x_{min},\ 1/4;\ x_{max},\ 3/4)$. In the experiment, the outcomes x_2 and x_3 vary in the range (x_{min}, x_{max}) and the lottery CE is determined. According to CPT, the CEs of L_4 should all fall between the CEs of L'_{2L} and L'_{2H} for any combination of x_2 and x_3.

In addition, the CEs of binary lotteries having various probabilities of winning are determined and used to estimate the CPT model, more specifically its probability weighting function. Predictions of this model with regard to CEs of three- and four-branch lotteries can then be tested empirically.

3. Experiment 1

3.1. Detailed design

Two payoff Sets were used with x_{max} = 300 zł[2] (Set 1) and x_{max} = 900 zł (Set 2). The lowest branch, x_{min}, assumed a value of 0 in both Sets. There were 18 binary lotteries $(0,\ 1-p;\ x_{max},\ p)$ constructed from a factorial design, in which p of winning x_{max} was either 0.01, 0.05, 0.10, 0.25, 0.5, 0.75, 0.9, 0.95, or 0.99, while x_{max} was either 300 or 900 zł.

There were 18 three-branch lotteries $(0, 1/3; x, 1/3;\ x_{max}, 1/3)$, where outcome x assumed values of 0, 15, 30, 75, 150, 225, 270, 285, and 300 zł in Set 1, and values of 0, 45, 90, 225, 450, 675, 810, 855, or 900 zł in Set 2, while x_{max} was either 300 or 900 zł.

Four-branch lotteries, $(x_{min}, 1/4; x_2,\ 1/4; x_3,\ 1/4; x_{max},\ 1/4)$ were constructed from all pairs of outcomes x_2 and x_3 (such that $x_2 \le x_3$) assuming values of 0, 30, 150, 270, and 300 zł in Set 1, and 0, 90, 450, 810, and 900 zł in Set 2. Lotteries (0, ¼; 100, ¼; 200, ¼; 300, ¼) and (0, ¼; 300, ¼; 600, ¼; 900, ¼) were added, making a total of 32 four-branch lotteries.

In total, there were 68 lotteries in the main experimental design: 18 two-branch (binary), 18 three-branch, and 32 four-branch. These were intermixed with 52 additional lotteries that can be considered fillers (these 3- and 4-branch lotteries involving branches of unequal probabilities of occurrence were constructed to test other hypotheses to be described in another paper). Altogether, 120 problems were presented to subjects.

3.2. Participants and incentives

There were 110 volunteers who were undergraduate economics

[1] To see this, observe that (\$12, 0.10) is worse than (\$12, 0.05; \$14, 0.05), and (\$90, 0.05; \$96, 0.85) is worse than (\$96, 0.90).

[2] The złoty is the Polish currency, \$1 ≈ 4 zł, although the purchasing power for basic goods is closer to parity.

students at the Warsaw School of Economics, and who were recruited by their supervisors. Their ages ranged from 18 to 25 years with a mean of 20.5 years, and 52% were women.

The experiment was conducted via the Internet. Six HTML forms (each with a different random order of the 120 problems) were prepared, and each participant was randomly assigned to one of the six forms at the start of the experiment. Participants first registered and familiarized themselves with the instructions online (see Appendix 3). They then responded to two sample problems. Based on an anticipated time of 40–50 min, participants received a 12-zł voucher redeemable at the campus cafeteria, but they were free to work at their own paces. The subjects were further incentivized for performance: They were informed that four of them would be randomly selected after the experiment to play a lottery for real cash prizes. The two who gave the lowest CEs for the randomly selected lottery received the amounts they quoted. The other two played that lottery and received real cash prizes resulting from independent plays.

3.3. Procedure for CE determination

The problems were described in a business-oriented way as risky ventures having 2, 3, or 4 scenarios with various probabilities of occurrence, each of which would have a monetary consequence. Fig. 1 shows two examples of how problems were displayed. In the first example (Problem 22), there were three possible scenarios yielding the outcomes 0, 75, and 300 zł, each with a probability of 33.3%. In the second example (Problem 61), there were two possible scenarios, each with an outcome of 450 zł and a probability of occurrence of 25%. The participants had to enter the value in the initially empty 100% box on the right that would make them indifferent between participating in a risky venture or accepting that sure sum of money.

3.4. Aggregating the data

Subjects' responses in experiments involving lottery CEs may be noisy, skewed and contain outliers. Moreover, people seem to respond with round numbers such as 10, 50, 250, 700 rather than 9, 52, 257, 691. Example histograms of CE responses obtained for 6 particular lotteries are presented in Fig. 2.

Because the mean is sensitive to outliers and the median to tied values, Wilcox (2012) recommends using the 20% trimmed mean for social science data. This statistic is the mean of the values remaining after the 20% with the smallest and the 20% with the largest values have been discarded (or "trimmed"). It is a compromise between the median and the mean and, according to Wilcox (2011, 2012), often outperforms more complex robust estimators when sampling from heavy-tailed distributions. The aggregated CE values thereby obtained are given in Appendix 1, and used in what follows.

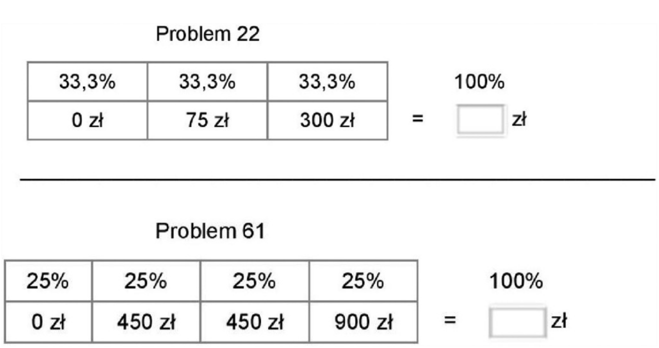

Fig. 1. Example problems involving a three- (top) and a four-branch (bottom) lottery.

3.5. The CPT model

The *CE* values are fitted to estimate the cumulative prospect theory (CPT) model:

$$v(CE) = \sum_{i=1}^{n} v(x_i)\left[w\left(\sum_{j=1}^{i} p_j\right) - w\left(\sum_{j=1}^{i-1} p_j\right)\right]$$

where x_i are lottery outcomes sorted in the decreasing order, p_i are the respective probabilities of x_i, $\sum_{j=1}^{i} p_j$ is the decumulative probability that an outcome in the lottery is equal to or exceeds x_i, and v denotes the value function. It is assumed that the value function is described using a power function $v(x) = x^{\alpha}$, and the decumulative probability weighting function is described using the two-parameter Lattimore et al. (1992) function:

$$w(P) = \frac{\delta\, P^{\gamma}}{\delta\, P^{\gamma} + (1 - P)^{\gamma}}$$

where $P = \sum_{j=1}^{i} p_j$.

3.6. Binary lotteries

The aggregated *CE* values for binary lotteries are presented in Fig. 3 as a function of probability p (orange dots). These values have been used to estimate the CPT models separately for Set 1 and Set 2 (their predictions are presented by the blue curves).

It can be seen that both estimated $CE(p)$ curves assume an inverse S-shape, as is usually the case in experiments involving binary lotteries (e.g. Tversky and Kahneman, 1992; Gonzales and Wu, 1999). The estimated γ parameters, both less than 1, indicate that the probability weighting functions are inverse S-shaped as well. The $CE(p)$ curves intersect the diagonal for $p = 0.46$ in Set 1 and for $p = 0.39$ in Set 2. The estimated *CE* values are therefore greater than the lottery expected value for $p = 1/3$ and $p = 1/4$, but lower than the lottery expected value for $p = 2/3$ and $p = 3/4$.

3.7. Three-branch lotteries

The aggregated *CE* values for three-branch lotteries are presented as a function of the middle outcome x in Fig. 4. The two horizontal dotted lines mark the *CE* values estimated by the CPT models for binary lotteries and for probabilities 1/3 and 2/3 (106.7 and 190.9 for Set 1, and 303.8 and 580.3 for Set 2). The lines define a band within which the *CEs* of three-branch lotteries should be located for all x values (this band is narrower than the one determined by lottery expected values, because the relation CE(p) presented in Fig. 3 is inverse S-shaped). More specifically, the leftmost point at $x = 0$ represents the lottery $L_3 = (0,\ 1/3;\ 0,\ 1/3;\ x_{max},\ 1/3)$ and, according to CPT, should be located on the lower line, while the rightmost point at $x = x_{max}$ represents the lottery $L_3 = (0,\ 1/3;\ x_{max},\ 1/3;\ x_{max},\ 1/3)$ and should be located on the upper line. However, as can be seen in both graphs, three *CE* values are located below the $CE(1/3)$ line, and four *CE* values are located above the $CE(2/3)$ line. Only 2 of the 9 *CE* values are within the band dictated by the results obtained for binary lotteries.

The aggregated *CE* values were used to estimate linear approximations, $CE = s\ x + a$, where s denotes the slope of x and a is a constant (see the graphs for the estimation details; the adjusted R-squared values are 0.99 and 0.98 respectively). It follows from CPT and linear approximation that the *CE* under- and overvaluation at both ends of the outcome range is about 15% compared to the $CE(1/3)$ and $CE(2/3)$ values estimated using binary lotteries. Importantly, the slope values s of the two curves (i.e. 0.437 and 0.445) exceed the probability of the middle branch x (i.e. 1/3). This indicates that the middle branch has a bigger impact on the lottery valuation than the probability of its occurrence would dictate, according to either EU or CPT as fit to binary lotteries.

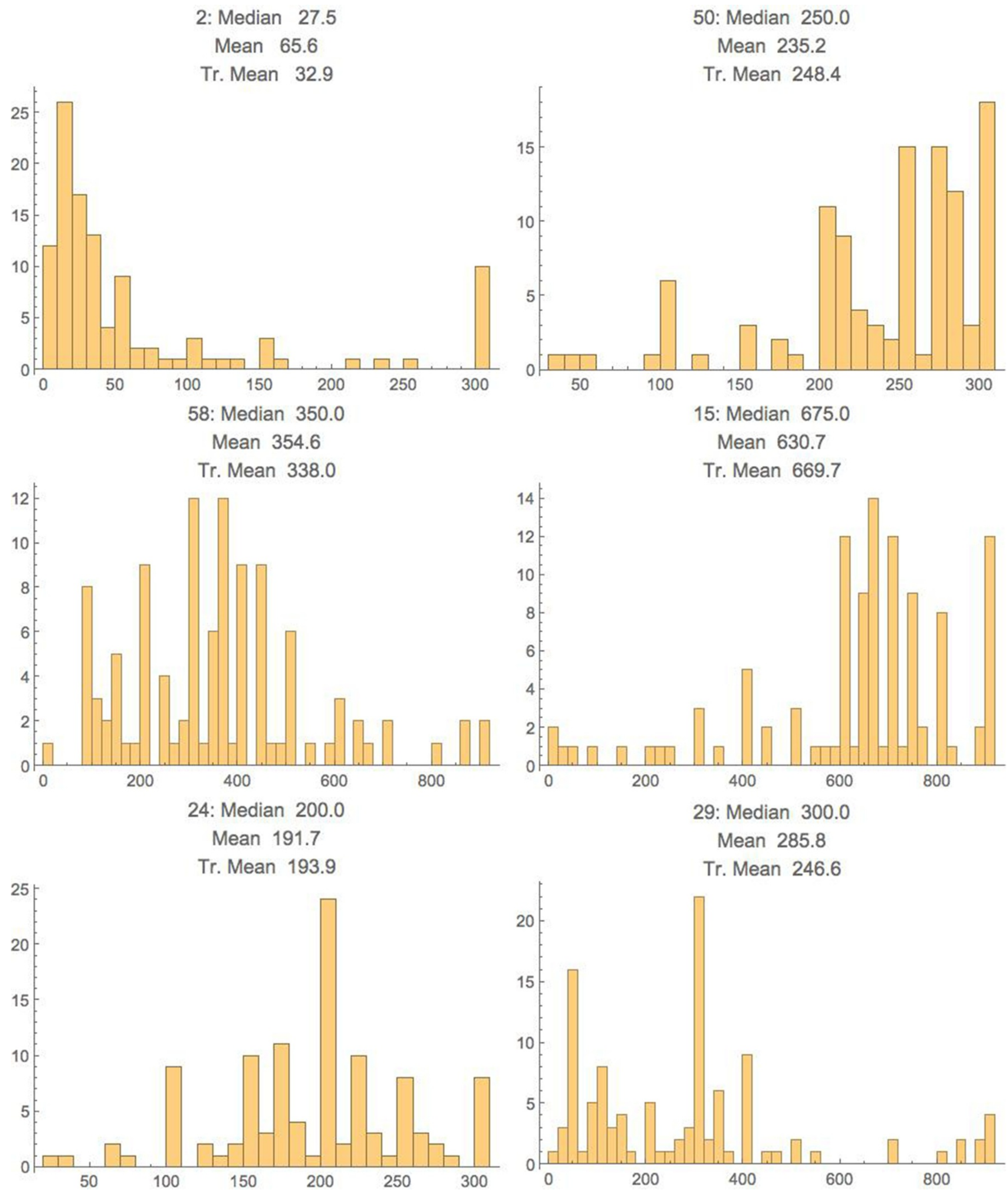

Fig. 2. Example histograms of CE responses for particular lotteries presented with mean, median and 20% trimmed mean values.

3.8. Violations and slopes

Note that violations of coalescing and of stochastic dominance observed in the present experiment are closely related. The finding that the lottery $L_3 = (0,\ 1/3;\ x,\ 1/3;\ x_{max},\ 1/3)$, where $x > 0$, is valued less than the lottery $L_{2L} = (0,\ 2/3;\ x_{max},\ 1/3)$ is a violation of stochastic dominance. However, for $x = 0$ the pattern is a violation of coalescing. Similarly, that the lottery L_3 is valued more than the lottery $L_{2H} = (0,\ 1/3;\ x_{max},\ 2/3)$ is a violation of stochastic dominance for $x < x_{max}$, and violation of coalescing for $x = x_{max}$.

Both violations are further interlinked with the estimated slope values for the middle outcome, x. In fact, starting points outside the band require high slope values for $0 < x < x_{max}$.

3.9. Four-branch lotteries

The aggregated *CE* values for four-branch lotteries are visualized as 3D surfaces, being functions of middle outcomes x_2 and $x_3 (x_2 \leq x_3)$, in Fig. 5.

Two planes are additionally presented. These assume values of *CE* (1/4) and *CE*(3/4) estimated by the CPT model for binary lotteries (85.7 and 213.9 for Set 1, and 234.0 and 652.2 for Set 2). As can be seen, the bottom-left corner of the *CE* surface is located below the *CE*(1/4) plane, whereas the top-right corner is located above the *CE*(3/4) plane.

The *CE* for four-branch lotteries have been used to estimate linear approximations: $CE = s_2 x_2 + s_3 x_3 + a$, where s_2 and s_3 denote the slopes

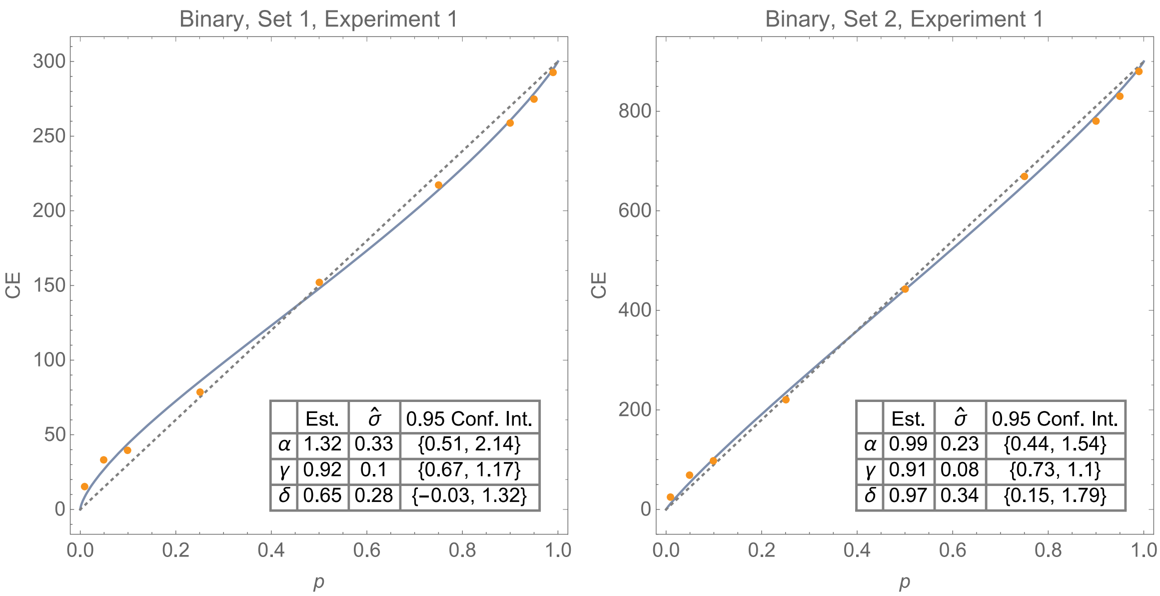

Fig. 3. CEs of binary lotteries presented as a function of the probability p of winning the greater payoff: Set 1 (left) and Set 2 (right). The points on the graphs (in orange) represent the aggregated ***CE***data for given probability values. The blue curves represent estimations using the CPT model. Estimation details: the standard errors and 0.95 confidence intervals of the estimated parameters are given in the tables on the graphs. (For interpretation of the references to colour in this figure legend, the reader is referred to the web version of this article.)

of x_2 and x_3 respectively, and a is a constant (see the graphs for the estimation details; the adjusted R-squared values are 0.99 and 0.98 respectively). It follows from the CPT and fitted linear equation that the under- and overvaluation of *CE* in both corners are again approximately 15%. As in the case of three-branch lotteries, the estimated slope values (0.304, 0.299 in Set 1, and 0.292, 0.314 in Set 2) exceed the probabilities of the outcomes of middle-ranked branches, x_2 and x_3 (i.e. ¼ and ¼). As the slope values of outcomes x_2 and x_3 are relative weights assigned to them during the lottery evaluation, it can be hypothesized that the middle outcomes receive psychologically higher weights than the probabilities of their occurrence.

As in the case of three-branch lotteries, both coalescing and stochastic dominance violations co-occur with high slope values for the middle outcomes x_2 and x_3. Points above and below the planes represent violations of coalescing and stochastic dominance. These crossover violations require either high slope values for outcomes x_2 and x_3, or they require the values of binary gambles (in this case, represented by the planes) to be spaced too closely together for corresponding probabilities, relative to middle branches of three- or four-branch gambles.

3.10. Self-contradiction of the CPT model

The observations presented so far are problematic for the CPT model which cannot predict violations of coalescing and stochastic dominance and, as it assumes an inverse S-shaped probability weighting function,

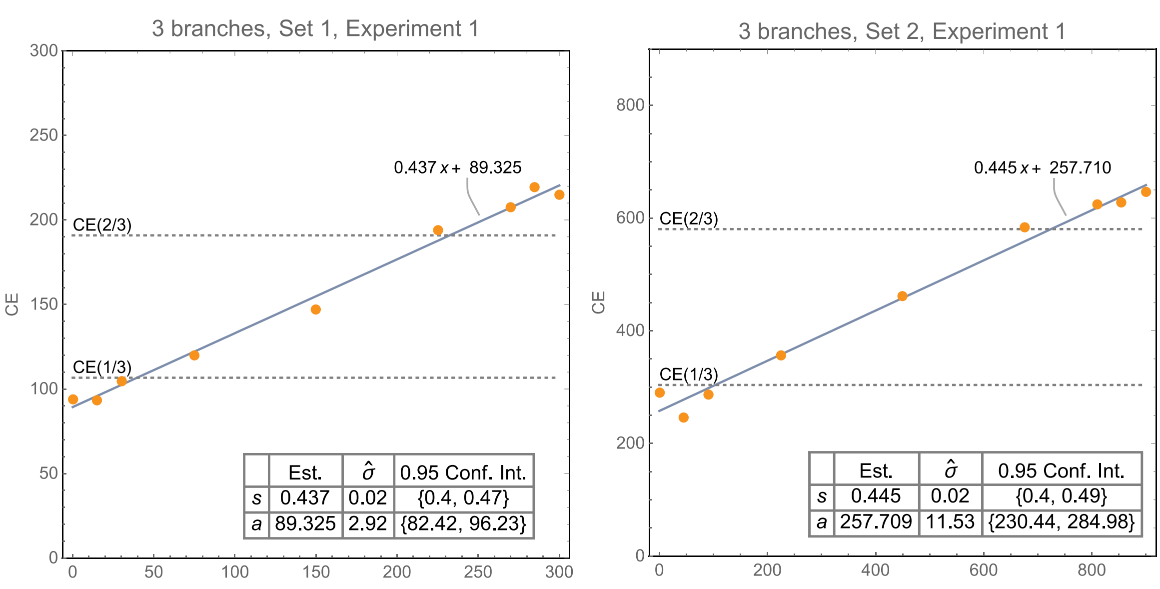

Fig. 4. *CE* values of three-branch lotteries presented as a function of x for two payoff Sets.

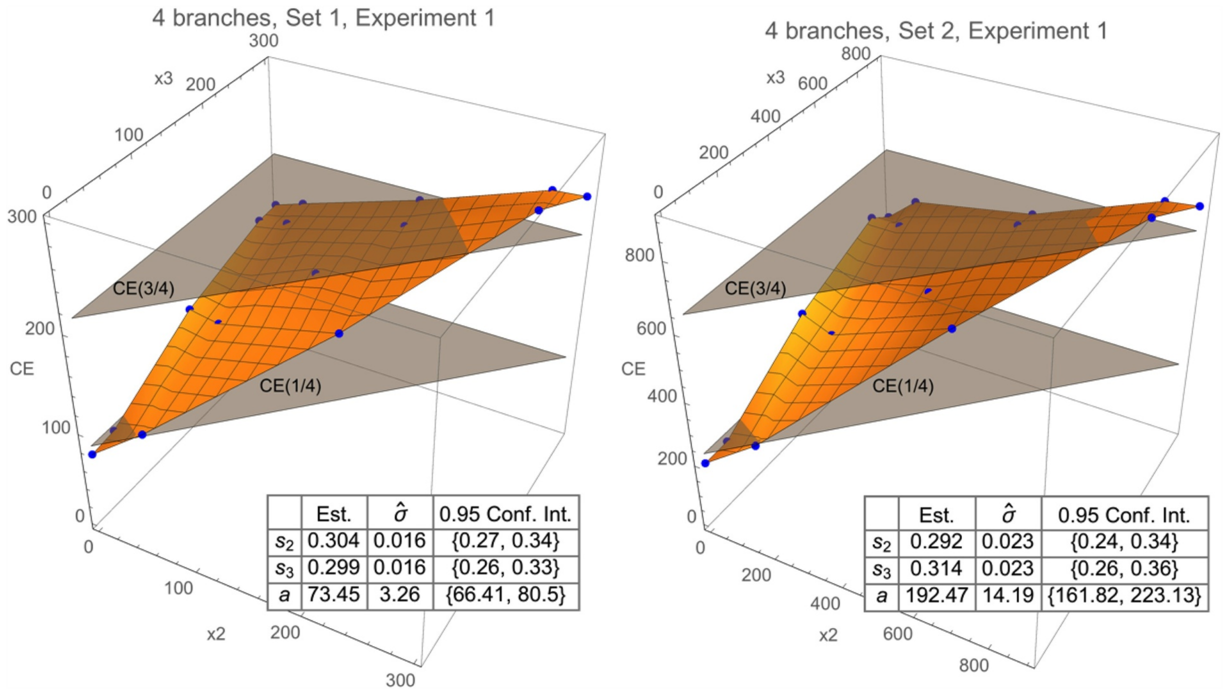

Fig. 5. *CE* surfaces as a function of $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$ for Set 1 (left) and Set 2 (right). The dots on the surface mark the lotteries involved in the experiment.

Table 1
Estimated parameter values of the CPT models: separately for Set 1, Set 2, and for all data. Standard errors are given in parentheses.

$\alpha = 1.043$	Set 1 γ	δ	Set 2 γ	δ	All data in both Sets γ	δ
2-branch	0.84 (0.03)	0.94 (0.04)	0.93 (0.03)	0.90 (0.03)	1.20 (0.04)	1.02 (0.02)
3-branch	1.37 (0.05)	1.03 (0.03)	1.40 (0.07)	0.99 (0.04)		
4-branch	1.29 (0.05)	1.20 (0.03)	1.30 (0.07)	1.06 (0.04)		

assigns lower weights to middle branches than to extreme ones. Portions of the observed data can, however, be described in CPT instead by an S-shaped probability weighting function.

To illustrate the contradiction in CPT, we first fit EU and CPT to all the available data. This fit yields a value function, which we then used to fit separate CPT models for two-, three-, and four-branch lotteries. These fits yield respective probability weighting functions, all with the same value function. The exponent of the value function is 1.042 (0.106) when estimated using the CPT model, and 1.043 (0.021) when estimated using the EU model. Estimated CPT model parameters

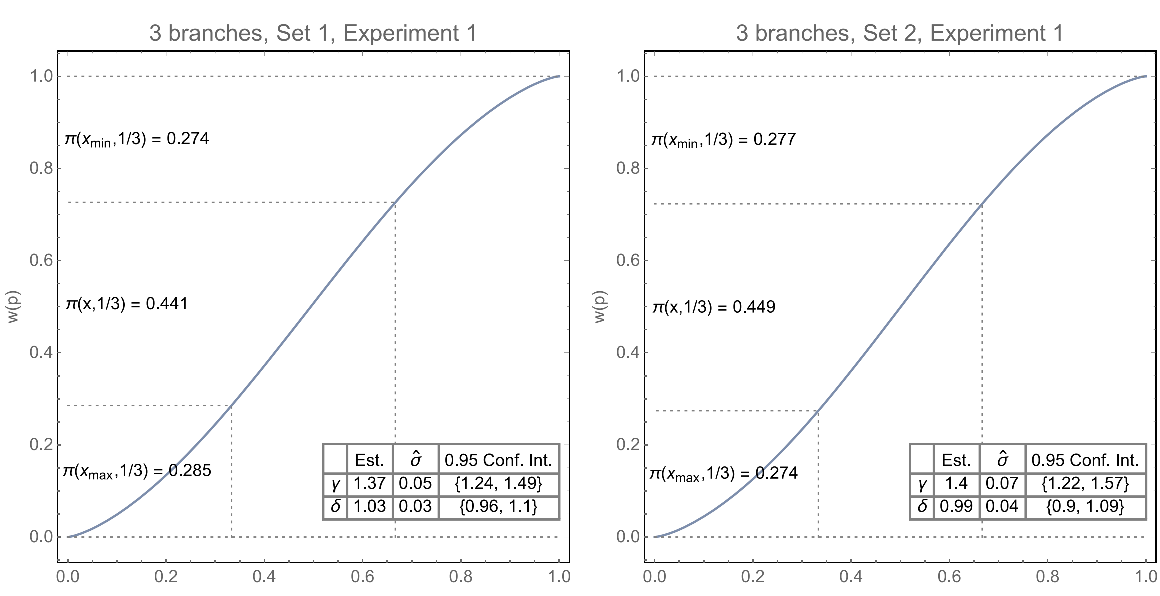

Fig. 6. The shapes of the probability weighting functions for Set 1 (left) and Set 2 (right) estimated using three-branch lotteries.

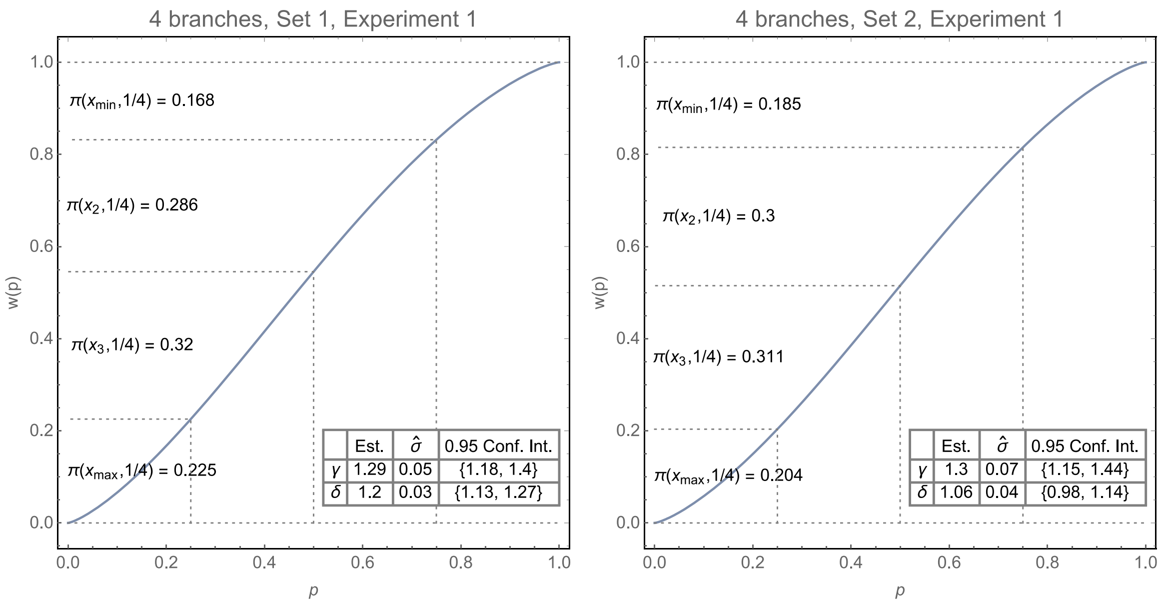

Fig. 7. The shapes of probability weighting functions for Set 1 (left) and Set 2 (right) derived using four-branch lotteries.

together with their standard errors are given in Table 1.

The shapes of probability weighting functions estimated using three-branch lotteries are presented in Fig. 6.

According to the functions derived for Set 1 and Set 2, the middle of three equally likely branches receives weights of 0.441 and 0.449, respectively (these weights are the differences between $w(2/3)$ and $w(1/3)$ according to the CPT model). These values are greater than 1/3 and agree with the slope values 0.437 and 0.445 in the linear approximations (Fig. 4). Note that, for three equally likely branches, the CPT model estimated using binary lotteries (with an inverse S-shaped probability weighting function) assigns a weight to the middle outcome x that is lower than the probability 1/3: 0.283 for Set 1 and 0.311 for Set 2. As shown, the weights are less than 1/3 for an inverse S-shape, and greater than 1/3 for an S-shape of the probability weighting function.

The shapes of CPT probability weighting functions obtained using four-branch lotteries are presented in Fig. 7. The weights assigned using the probability weighting functions fit to four-branch lotteries are 0.286 and 0.320 for Set 1, and 0.300 and 0.311 for Set 2, which are all greater than 1/4. These values are in accordance with the slope values 0.304, 0.299, 0.292, and 0.314 in the linear approximations of Fig. 5. If the probability weighting functions estimated for binary lotteries from CPT were used, then the weights would be: 0.219 and 0.213 for Set 1, and 0.240 and 0.229 for Set 2, which are all lower than the probability 1/4.

Finally, the shapes of estimated probability weighting functions for two-, three-, and four-branches are presented together in Fig. 8.

The probability weighting function of CPT is inverse S-shaped for binary lotteries (in blue, the parameter responsible for the curvature $\gamma < 1$), and S-shaped for three- and four-branch lotteries (in yellow and green, respectively, $\gamma > 1$) in both Sets. The S-shape clearly contradicts the inverse S-shape estimated in this experiment for binary lotteries, as well those reported in other experiments with binary lotteries (e.g. Tversky and Kahneman, 1992; Gonzales and Wu, 1999). The pattern observed in this experiment for three- and four-branch lotteries, more

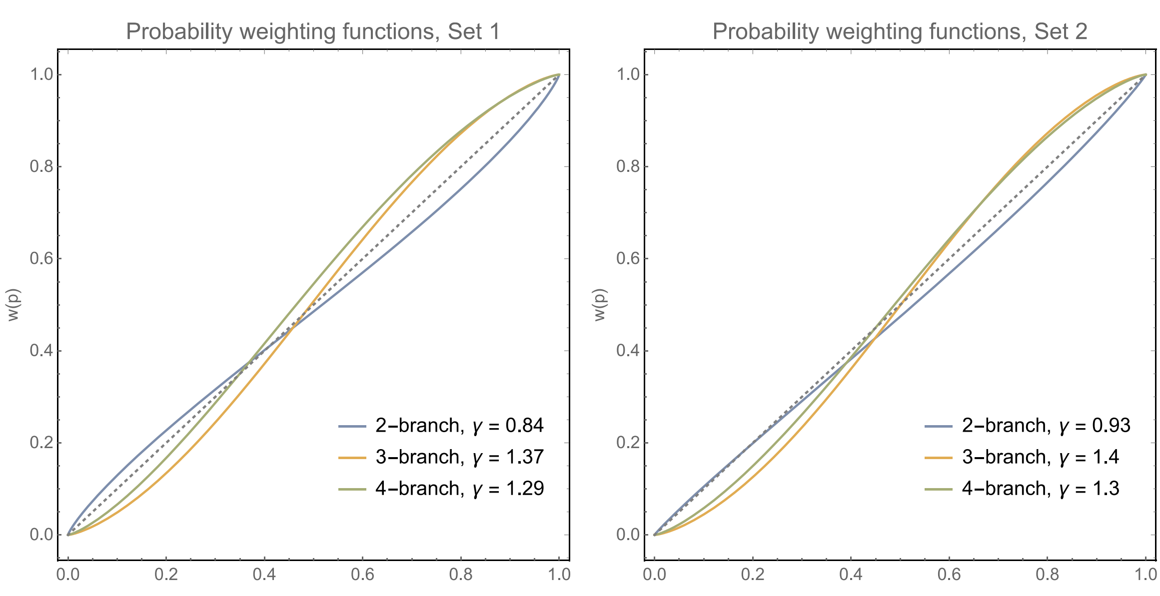

Fig. 8. Probability weighting functions in the CPT models derived separately for two-, three-, and four-branch lotteries, and for Set 1 (left) and Set 2 (right). The power coefficient of the value function was estimated from all data and then held constant at 1.043 when fitting the data separately.

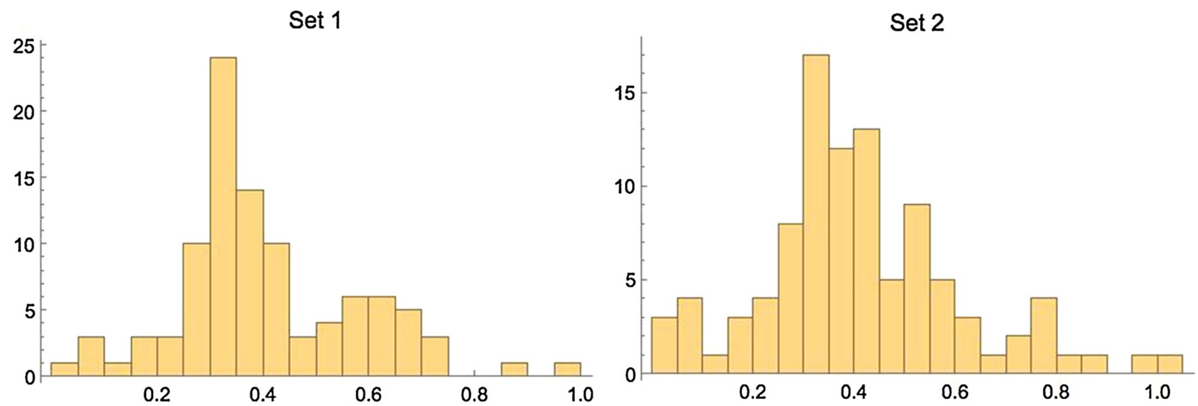

Fig. 9. Histograms of the estimated slope values s for Set 1 (left) and Set 2 (right) in three-branch lotteries.

specifically, high weights assigned to the middle outcome(s), requires an S-shaped probability weighting function, contrary to the pattern observed for binary lotteries, which requires an inverse S-shaped probability weighting function. It should be emphasized that these self-contradictions of the weighting functions (Fig. 8) are obtained for the same subjects in the same experiment.

Note also that the CPT probability weighting function estimated for all data is also S-shaped: its γ parameter, characterizing the curvature, exceeds 1 (see Table 1). Its overall value of 1.20 is a best-fit compromise of the γ values describing two-branch lotteries (0.84 and 0.93), three-branch lotteries (1.37 and 1.40), and four-branch lotteries (1.29 and 1.30). Because there were fewer two-branch lotteries (18) than the sum of three-branch and four-branch lotteries (18 + 32) in the study, the resulting probability weighting function for all the data resembles more closely the S-shaped curves than the inverse-S observed with two-branch lotteries.

3.11. An analysis on the individual level

The results obtained using aggregated data were also found to be descriptive of individual data. We fit each person's data to individual CPT models for binary lotteries and linear approximations for three- and four-branch lotteries. In three branch-lotteries, the best-fit slope of CE as a function of the middle outcome is an estimate of weight that can be compared with weight estimated by CPT from binary lotteries (keep in mind that the best-fit value function for CPT was nearly linear in this study). According to inverse-S weighting functions estimated from binary lotteries, the middle branch should have a weight less than 1/3. However, 63 individuals had estimated slopes in the linear approximation greater than 1/3 in both Set 1 and in Set 2. An even greater majority had estimated slopes greater than weights estimated individually using CPT fit to binary gambles: 89 (80.9% of all 110 participants) in Set 1 and 81 (73.6%) in Set 2. Histograms of the estimated slope values s are presented in Fig. 9 (not shown are estimated slopes for 12 subjects who had nonpositive slopes).

Histograms of the estimated slope values s_2 and s_3 values in four-branch lotteries are presented in Fig. 10 (not shown are estimates from 24 subjects whose estimates were nonpositive).

Many individuals have estimated slopes greater than 1/4: 56 and 50 subjects in Set 1, and 45 and 55 subjects in Set 2. For the majority of subjects (80 and 78 for Set 1, and 70 and 72 for Set 2), the slope values are greater than the respective weights estimated from binary lotteries in their individual CPT models. Thus, individual analysis of three- and four-branch data show that the majority of individuals systematically violate CPT, if the probability weighting function is supposed to be the same for two-, three-, and four-branch lotteries.

4. Experiment 2

4.1. Detailed design

The payoff values in Experiment 2 were in \$, with x_{min} = \$5 and x_{max} = \$95.

There were 9 three-branch lotteries of the form (\$5, x, \$95), with nine levels of x = \$10, \$20, \$30, \$40, \$50, \$60, \$70, \$80, or \$90. Each three-branch lottery was presented twice, resulting in 18 problems to be solved.

There were 81 four-branch lotteries (\$5, b_2, b_3, \$95) constructed from a 9 × 9, $b_2 \times b_3$, factorial design, where b_2 and b_3 denote the values of the second and the third possible prizes, respectively, which could take the same 9 values. Thus, subjects received both (\$5, \$20, \$60, \$95) and (\$5, \$60, \$20, \$95), for example, but one (\$5, \$20, \$20, \$95) only. Note that in this set-up, the ranked outcomes x_2 and x_3 are respectively the lower and the higher of the b_2 and $b_3$values (as we assume that $x_2 \le x_3$); hence, $x_2 = Min(b_2, b_3)$ and $x_3 = Max(b_2, b_3)$. The lottery (\$5, \$45, \$55, \$95) was added to this set. Combining these 9 × 9 = 81 four-branch trials with 18 three-branch trials, plus this added lottery, there were 100 problems in the experiment, which were intermixed randomly with up to 33 "filler" trials and presented in random order, following the instructions and warmup trials. The "fillers" were trials with from two to ten equally likely prize values, where the prize values were between \$5 and \$95.

4.2. Participants

Seventy-six subjects took part in the experiment. Sixty-two of them were undergraduate psychology students at the California State University, Fullerton, of whom 65% were women. Fourteen were undergraduate economics students at the Warsaw School of Economics, and 57% of that group were women.

The experiment was conducted via the Internet and the subjects could respond at their convenience. The participants first registered and familiarized themselves with the instructions online (see Appendix 4). They were then required to solve four sample problems, and could then work at their own paces for the 100 experimental and "filler" trials.

4.3. CE determination

The lotteries were displayed as in the following example:

(\$5, \$70, \$95) Cash Value = \$

The participants had to state the value that would make them indifferent between participating in a lottery and accepting a sure sum of money.

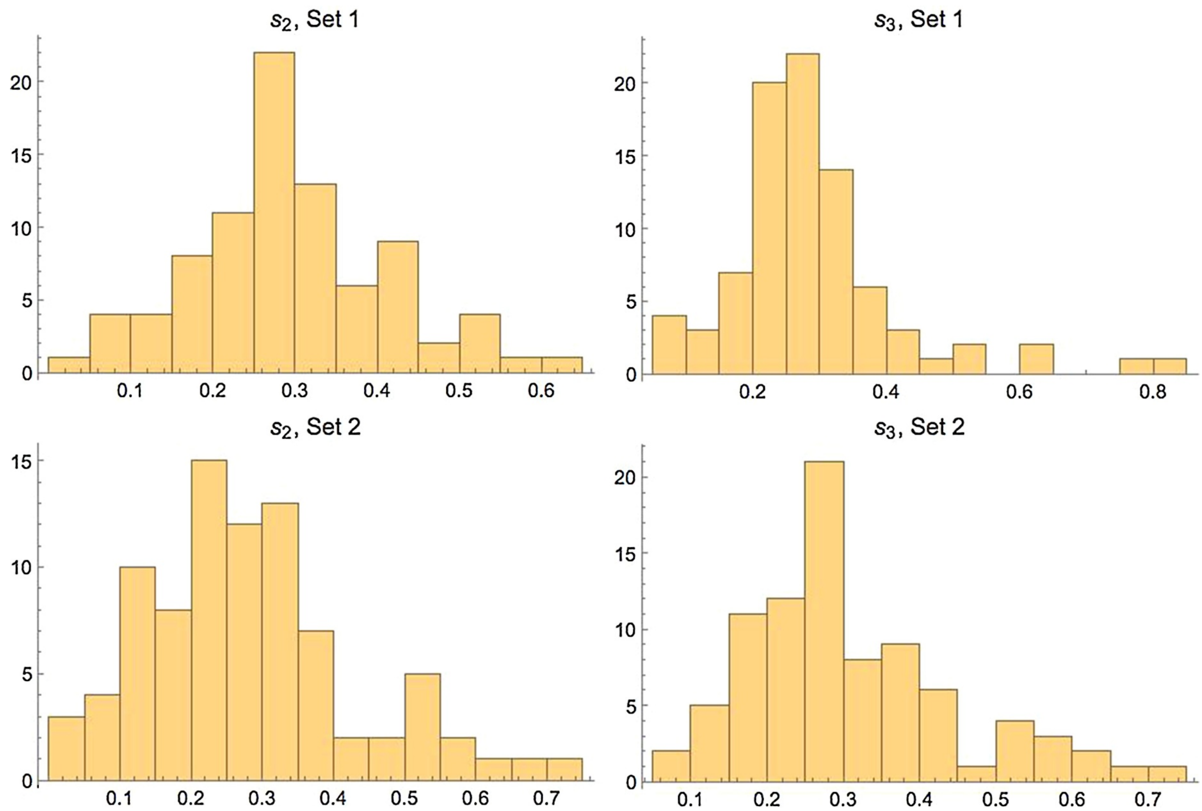

Fig. 10. Histograms of estimated slope values in linear models for four-branch lotteries.

4.4. Three-branch lotteries

The individual CE data were aggregated using a 20%-trimmed-mean (see Appendix 2 for the results). The aggregated CE values for three-branch lotteries are presented as a function of the middle outcome *x* in Fig. 11.

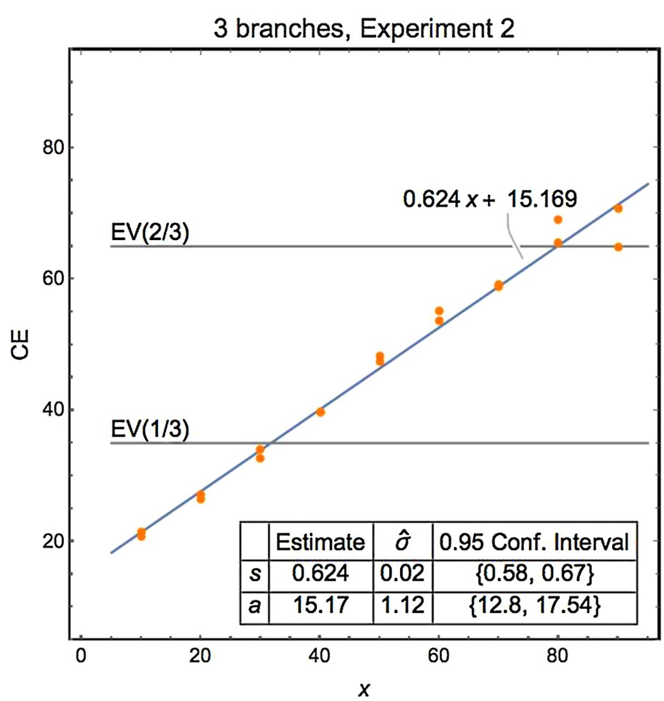

Fig. 11. Aggregated *CE* values for three-branch lotteries presented as a function of *x*.

The two horizontal solid lines mark the *CE* values $35 and $65, which are expected values of binary lottery ($5, 1-*p*; $95, *p*) with probabilities *p* of ⅓ and ⅔. If an inverse-S probability weighting function were used to calculate CEs, the band of supposed CE values would be even narrower. As seen, aggregated *CE* values are located below the EV(⅓) line for $x \le \$30$, and above or on the EV(⅔) line for $x \ge \$80$. Note that two CE points are given for each *x* value, as each lottery was presented twice. Thus, the average values for $x \ge \$80$ are greater than the lottery expected value for $p = ⅔$.

The *CE* values have been used to estimate a linear approximation: $CE = s\ x + a$ (see the graph for details; the adjusted *R*-squared = 0.98). It follows from the equation that calculated CEs of ($5, $5, $95) = $18.3 and ($5, $95, $95) = $74.5 are 47.8% less, and 14.6% greater than their expected values ($35 and $65, respectively). The curve slope value of 0.624 is much greater than 1/3, the probability of the middle outcome *x*. This overweighting of the middle outcome is even more extreme than found in Experiment 1, where the slopes were 0.437 and 0.445 in Sets 1 and 2, respectively.

Fig. 12 shows a separate examination of Fig. 11 for Californians (psychology students) and Poles (economics students).

The slope values (0.651 and 0.492, respectively) both exceed 1/3 and are even higher for Californians than Poles. The level of mathematical education or some other confounded variable, rather than country of origin, may be the reason for the differences; but both subgroups show the same qualitative crossover.

4.5. Four-branch lotteries

The aggregated *CE* values for four-branch lotteries are visualized in Fig. 13 as a function of middle branches b_2 and b_3 (on the left). The graph on the right presents the CE surface as a function of $x_2 = Min(b_2,\ b_3)$ and $x_3 = Max(b_2, b_3)$, where CE is the average for lotteries (5, b_2, b_3, 95) and (5, b_3, b_2, 95).

Two planes of constant values of $27.5 and $72.5, expected values

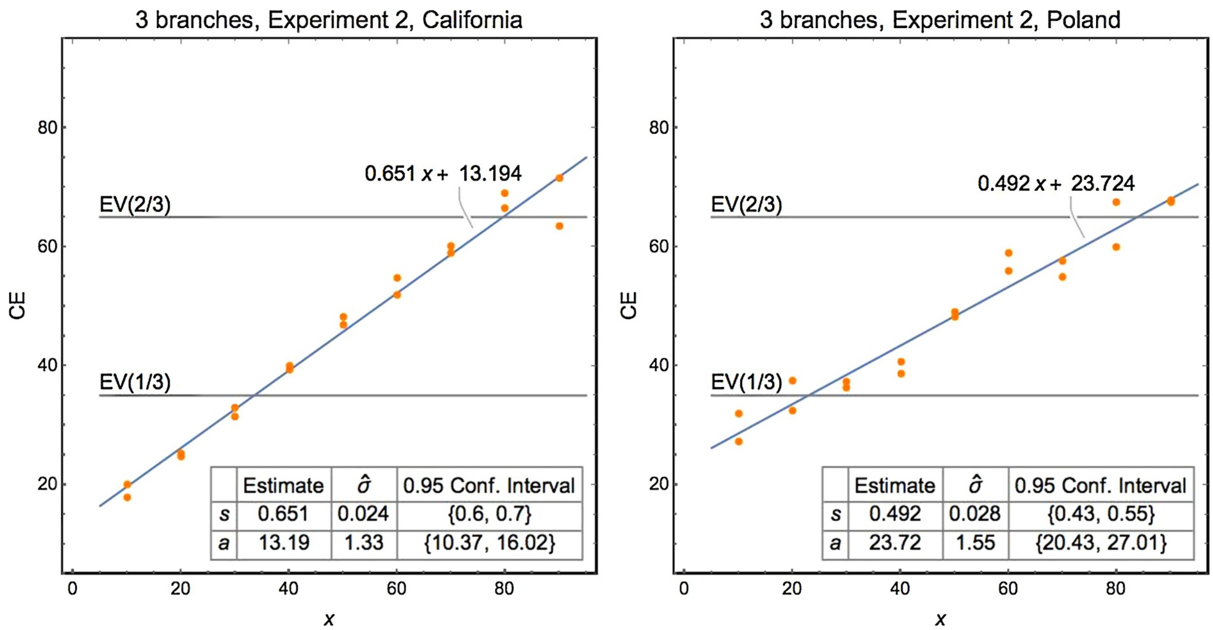

Fig. 12. *CE* values of three-branch lotteries presented as a function of *x* for subjects from California (left), and subjects from Poland (right).

of ($5, 3/4, $95, 1/4) and ($5, 1/4; $95, 3/4), respectively are shown in the figure. The bottom-left corner of the CE surface is located below the EV(1/4) plane, and the top-right corner is above the EV(3/4) plane. The *CE* surface below and above the CE(1/4) and CE(3/4) planes would have been larger if an inverse S-shaped probability weighting function had been used to calculate CE values for those binary lotteries.

We fit the linear approximation: $CE = s_2x_2 + s_3x_3 + a$ (see Fig. 13 for details; the adjusted R-squared = 0.917). The calculated CE is $19.5 for $x_2 = x_3 =$ $5, which is 29.1% less than the lottery expected value ($27.5), whereas the CE is $77.8 for $x_2 = x_3 =$ $95, which is 7.3% more than the lottery expected value ($72.5). The estimated slope values for x_2 and x_3 are 0.428 and 0.247, respectively, so the second-lowest outcome received greater weight than the second-highest. The sum of both weights assigned to middle outcomes is 0.675, leaving a remainder sum weight of 0.325 for minimum and maximum outcomes. This sum for middle outcomes (0.675) exceeds the sum of objective probability (0.5), as in Experiment 1, and is even more extreme than the corresponding values from Experiment 1 (0.603 and 0.606 for Sets 1 and 2).

4.6. The CPT model

CPT models were fit as follows: First, the value function and probability weighting function were estimated using all available data for Experiment 2; then separate CPT models for three- and four-branch lotteries were estimated (using the value function estimated from all data) to examine whether the probability weighting functions differ

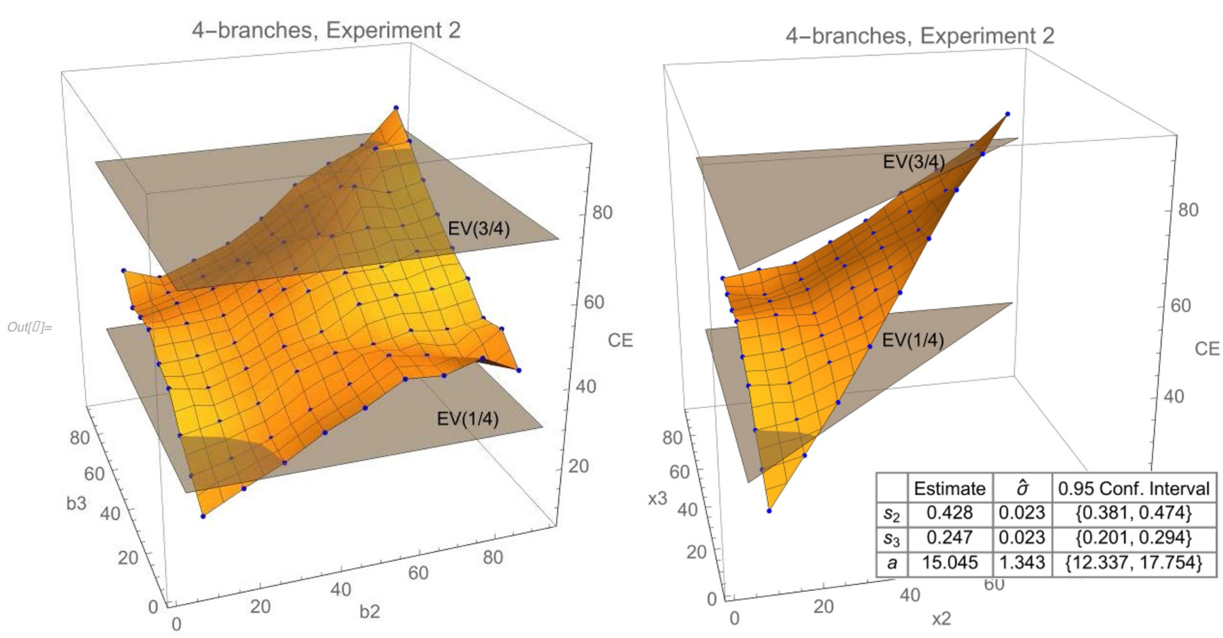

Fig. 13. *CE* surface as a function of $\boldsymbol{b}_2$ and $\boldsymbol{b}_3$(left), and $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$(right) for four-branch lotteries in Experiment 2. The dots on the surface mark the lotteries involved in the experiment.

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Table 2
Estimated parameter values of the CPT models: for all data, and separately for three- and four-branch lotteries. Standard errors are given in parentheses.

α = 1.32 (0.21)	γ	δ
All data	1.71 (0.09)	0.59 (0.10)
3-branch	2.22 (0.11)	0.58 (0.03)
4-branch	1.53 (0.07)	0.60 (0.01)

systematically for these cases. The exponent of the value function estimated from all the data is α= 1.325 (standard error = 0.212). Estimated CPT model parameters for separate fits, together with their standard errors, are given in Table 2.

Fig. 14 shows the estimated probability weighting functions of CPT for middle branches in three- and four-branch lotteries. These are S-shaped, rather than inverse-S.

The weight assigned to the middle branch, according to the CPT model for three-branch lotteries, is 0.620, close to the value of 0.624 estimated in the linear approximation in Fig. 11 (left).

The weights assigned to ranked outcomes x_2 and x_3, according to the CPT model for four-branch lotteries, are 0.388 and 0.274, respectively (compared to slope values 0.428 and 0.247 of the linear approximation in Fig. 13) Their sum in both models (CPT and linear) is however similar (0.662 vs. 0.675).

4.7. An analysis on the individual level

Linear approximations for three- and four-branch lotteries were fitted separately for each subject. There were 49 individuals who had slopes greater than 1/3 in three-branch gambles: (64% of all 76 participants). Histograms of the estimated slopes are shown in Fig. 15 (left) (not shown are estimates from 3 subjects whose estimates were nonpositive).

For slopes estimated from four-branch lotteries: 48 subjects had $s_2 > 1/4$; 33 subjects had $s_3 > 1/4$; and for 43 subjects the sum of $s_2 + s_3 > 1/2$. The right panel of Fig. 15 shows a histogram of the sums of estimated slopes s_2 and s_3, excluding estimates from 15 subjects who had nonpositive slopes. In sum, the overweighting of middle outcomes found at the aggregate level is also characteristic of the majority of individual participants' data.

5. Discussion

This paper reports new examples of violation of coalescing and stochastic dominance, as well as evidence that middle outcomes apparently being overweighted (relative to extreme ones) in judgments of certainty equivalents. The main conclusions extend and confirm previous findings.

The results of this paper cannot be described by CPT or EU, because those theories cannot explain violations of coalescing or stochastic dominance. We found, for example, that lotteries (0, 1/3; 300, 2/3) and (0, 1/3; 300, 1/3; 300, 1/3), which are objectively equivalent receive different evaluations, with the three-branch lottery judged higher than the equivalent two-branch lottery. Further, certain lotteries with equally likely branches (0, 285, 300), (0, 270, 300), and (0, 225, 300), are valued more than the binary lottery (0, 1/3; 300, 2/3), despite being stochastically dominated by it. The opposite is observed when the middle branch value is low, i.e. certain three-branch lotteries are valued less than a comparable binary lottery, despite being objectively dominant over it.

It is not always the case that preference orders obtained from judgments of the value or attractiveness of lotteries and from choices between lotteries are the same (e.g. Schmidt and Trautmann, 2014). Cases where lottery A is evaluated higher than B but people choose B over A are called "preference reversals" and several theories have been developed and tested to describe them (e.g., Mellers et al., 1992). However, in this paper we have a case where similar results are observed in both judgment and choice: violations of coalescing and stochastic dominance like those described here for judgments have also been observed in many studies of choice (Birnbaum, 2008). When the same phenomena are observed in both judgment and choice, it seems

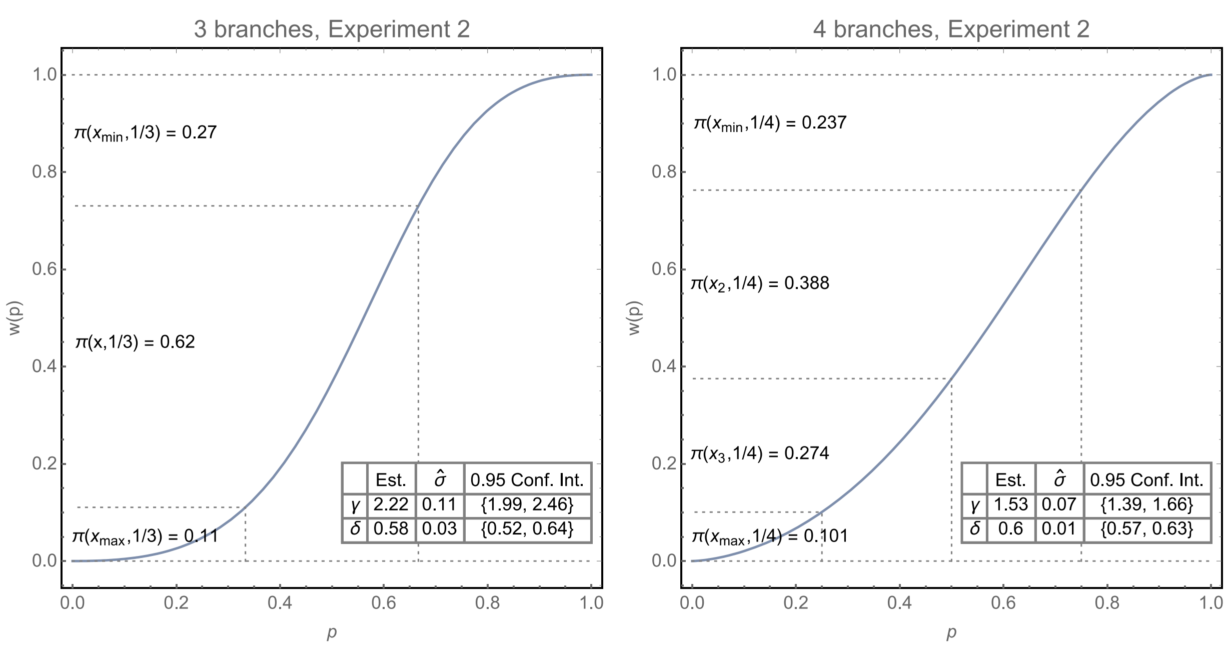

Fig. 14. Probability weighting functions estimated separately for 3-branches (left) and 4-branches (right).

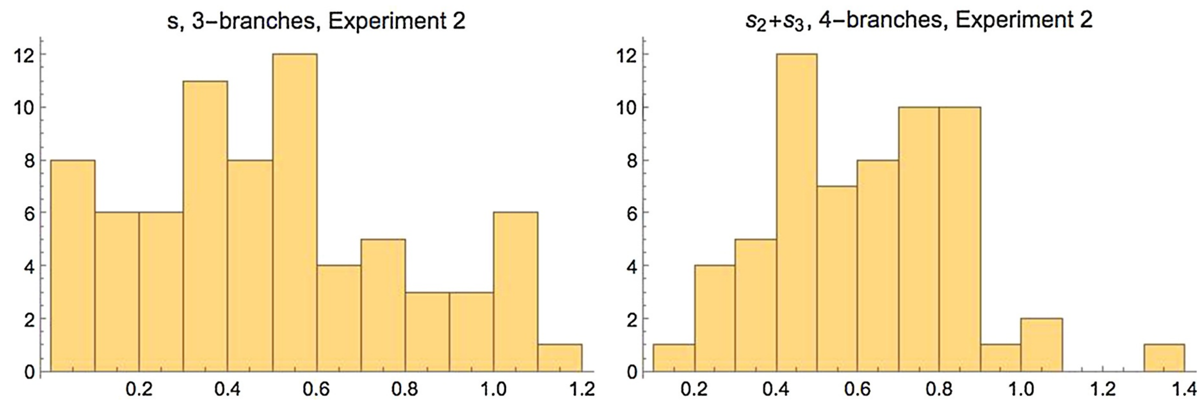

Fig. 15. Histograms of individual slope values for 3-branch lotteries (left) and of the sum of slopes s_2 and s_3 for four-branch lotteries (right).

reasonable to assume that the same kind of mechanism for evaluation of the lotteries applies in both cases, though different parameters may be involved.

Our finding that middle branches receive greater weight than extreme ones is consistent with results with judgments of selling prices of gambles (Birnbaum and Beeghley, 1997), but it is not necessarily consistent with studies of choices among gambles. In a general TAX model (Birnbaum, 2004a), transfers of weights from branch to branch are free, so it is possible for the middle branch to have the highest weight. However, in the special TAX model (Birnbaum, 2008), the attention transfers are all equal, so the middle branch of three equally likely branches is intermediate in weight. Despite this, the special TAX model can account for a variety of phenomena in choice data, including violations of stochastic dominance and coalescing, without assuming greatest weight for the middle branch.

When we attempted to fit the CPT model to our data, the solutions led to self-contradiction. Binary lotteries could be fit with a probability weighting function with the usual inverse-S shape, observed in previous studies, such as in Tversky and Kahneman (1992). However, when we fit the CPT weighting function separately to lotteries with three or four branches, we estimated that middle branches have greater relative weight than their probabilities, leading to a weighting function with the opposite shape; i.e., S-shaped. Birnbaum and Navarrete (1998) also fit CPT to show that judgments of binary lotteries and three-branch lotteries lead in CPT to contradictory probability weighting functions, similar to our findings, except in a study of choice instead of judgment. It was found in many studies of choice, starting with Birnbaum and McIntosh (1996), that the middle branch has greater weight than allowed by *any* inverse-S weighting function of CPT.

The finding that CPT leads to contradictory probability weighting functions is taken as evidence that the model is wrong, so parameters derived using binary lotteries should not be expected to predict results with multi-branch lotteries. Our conclusions may seem surprising, as many people once considered CPT to be a well-established model. But keep in mind that Tversky and Kahneman (1992) fit the CPT model to binary lotteries and subsequent evidence for the inverse-S weighting function in CPT came from binary lotteries (e.g. Gonzales and Wu, 1999). Research involving more outcomes and better experimental designs have led to problems for the CPT model. For instance, Kontek (2018) shows that the CPT model is ranked only fourth in the ranking of models fitting the data from two- and three- branch lotteries in the Marschak–Machina triangle.

Dozens of other studies falling outside the Marschak–Machina triangle have confirmed direct contradictions in CPT (Birnbaum, 2004b, 2008; Birnbaum and Bahra, 2012). For instance, violations of first order stochastic dominance have been observed in pairs of three branch lotteries in choice (Birnbaum and Navarrete, 1998), and in judgment (Birnbaum et al., 2016). That means that even if we tried to salvage CPT by allowing a different weighting function for three-branch lotteries, we could not explain those violations of stochastic dominance.

The finding that middle branches receive weight exceeding their probabilities stands in contrast not only to CPT with its inverse-S weighting function, but also to other decision models, such as the priority heuristic (Brandstätter et al., 2006), which assumes that people ignore middle branches.

If we reject CPT, we are able to describe our results using an averaging model with configural weights (Birnbaum, 1997) in which the weights of branches depend on their probability, ranks, and number of branches:

$$CE = \frac{\sum_{i=1}^{n} r_{i,n} w(p_i)\, x_i}{\sum_{i=1}^{n} r_{i,n} w(p_i)}$$

where $w(p_i)$ is the weight of the branch's probability (not decumulative probability), and $r_{i,\ n}$ is the configural weight of branch ranked i in a lottery with n distinct branches. As noted by Birnbaum (1997), when $w(p) = p^{\gamma}$, this model can approximate CPT for binary gambles (allowing a nonlinear utility function would make the approximation even closer). When $\gamma < 1$, even with all of the $r_{i,n} = 1$, this model implies an inverse-S relation between CE(x, p; y) and p; and when $\gamma < 1$, this model also predicts violations of coalescing and stochastic dominance of the type we observed.

Thus, the contradictions we observed in CPT are consistent with an averaging model with configural weights, in which the weight of a branch is a negatively accelerated function of branch probability. Further, we can describe other aspects of our data by the assumption that weights are affected by the ranks of the outcomes on discrete branches (not decumulative probability). One can interpret the slopes estimated in our linear approximations to effects of the outcomes of middle branches to be estimates related to the $r_{i,\ n}$ in the above model. According to our data, the relative weights of the middle-ranked branches in two and four-branch gambles are greater than their probabilities.

Such a RAM (Rank-affected multiplicative weights model) is however not fully satisfying because it requires separate configural weights for each ranked branch and number of outcomes. Moreover, it does not explain why the weights would be higher for middle branches, for example. What is sought is a simpler, configural or contextual theory, which we think might take the form of an extension of Parducci's (1965) range-frequency theory, that could provide a simpler account of the phenomena that we now describe by empirically estimated configural weights.

7. Appendices

7.1. Appendix 1: Aggregated CE values obtained in Experiment 1

Binary lotteries (p - probability of obtaining x_{max}, $x_{min} = 0$)

p	**0.01**	**0.05**	**0.1**	**0.25**	**0.5**	**0.75**	**0.9**	**0.95**	**0.99**
CE: $x_{max} = 300$	15.4	32.9	39.8	78.4	152.1	217.3	258.9	275.0	292.7
CE: $x_{max} = 900$	24.0	69.3	97.3	220.6	442.6	669.7	781.0	829.9	880.6

Three-branch lotteries ($x_{min} = 0, x_{max} = 300$)

x	**0**	**15**	**30**	**75**	**150**	**225**	**270**	**285**	**300**
CE: $x_{max} = 300$	93.6	93.4	104.6	119.8	146.9	193.9	207.6	219.4	214.7

Three-branch lotteries ($x_{min} = 0$, $x_{max} = 900$)

x	**0**	**45**	**90**	**225**	**450**	**675**	**810**	**855**	**900**
CE: $x_{max} = 900$	290.2	246.6	286.3	357.1	461.3	584.3	623.6	627.0	646.5

Four-branch lotteries ($x_{min} = 0$, $x_{max} = 300$)

CE: $x_2 \| x_3$	**0**	**30**	**150**	**200**	**270**	**300**
0	76.4	79.5	130		157.7	159.3
30		87.5	124.7		163.2	169
100				170.3		
150			155.7		195.1	205.8
270					244.5	248.4
300						250.6

Four-branch lotteries ($x_{min} = 0$, $x_{max} = 900$)

CE: $x_2 \| x_3$	**0**	**90**	**450**	**600**	**810**	**900**
0	202.7	205.1	376.2		480.9	436.6
90		227.6	338.0		480.1	511.0
300				452.0		
450			482.2		586.4	572.8
810					711.7	712.3
900						723.1

7.2. Appendix 2: Aggregated CE values obtained in Experiment 2

Three-branch lotteries ($x_{min} = 5$, $x_{max} = 95$)

X	**10**	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**
CE1	20.7	26.5	32.6	39.8	47.5	53.7	58.8	65.6	64.8
CE2	21.4	27.2	34.0	39.7	48.3	55.2	59.2	69.1	70.8

Four-branch lotteries ($x_{min} = 5$, $x_{max} = 95$)

CE: $x_2 \| x_3$	**10**	**20**	**30**	**40**	**50**	**55**	**60**	**70**	**80**	**90**
10	18.0	22.5	27.5	34.7	36.4		41.0	40.1	38.7	45.0
20	22.9	25.0	29.3	33.0	36.8		41.0	42.3	41.2	41.8
30	27.4	28.4	32.1	36.3	38.6		41.6	44.7	44.3	45.1
40	33.1	33.7	38.6	40.5	44.5		46.7	50.2	47.4	48.5
45						48.3				
50	37.4	38.4	41.7	42.8	49.2		51.2	54.5	53.2	54.2
60	42.8	44.0	42.4	48.4	52.5		58.4	55.4	59.6	62.2
70	42.1	42.5	44.4	49.3	52.7		59.3	66.9	63.7	67.0
80	44.8	40.0	43.3	49.5	55.5		58.7	64.1	72.7	70.8
90	40.2	46.2	44.8	51.0	55.3		60.6	66.5	73.8	80.0

7.3. Appendix 3: Instruction used in Experiment 1 (a translation from Polish)

You have a choice to either invest in a risky venture or earn a sure sum of money.
The risky venture may have 2, 3, or 4 scenarios. These may be:

a) a pessimistic and an optimistic scenario (when two scenarios are present);
b) a pessimistic, a neutral, and an optimistic scenario (when three scenarios are present);
c) a very pessimistic, a moderately pessimistic, a moderately optimistic, and a very optimistic scenario (when four scenarios are present).

The scenarios, from the most pessimistic to the most optimistic, are presented in subsequent columns in a table.
The probabilities of occurrence for each scenario are given in the upper row of the table.
The amounts you can earn in each scenario are given in the lower row of the table.
If you invest in a risky venture, your payoff will depend on which scenario occurs. The risk lies in the fact that you have no influence over this.
On the other hand, you can avoid taking part in a risky venture, and earn a sure sum of money (i.e. with 100% certainty) whatever the scenario (e.g. you can put your money into a savings account).
State the sure sum of money that would make you indifferent between accepting it and taking part in a risky venture, i.e. so that it would not matter to you whether you received this sure sum or took part in the venture.
Example 1

Problem xx

33,3%	33,3%	33,3%		100%
0 zł	100 zł	300 zł	=	[ ] zł

According to the table, you will earn 0 zł in the pessimistic scenario, 100 zł in the neutral scenario, and 300 zł in the optimistic scenario. Each scenario has a 33.3% chance of occurrence.

a) Think of the sure sum of money that would make you indifferent between receiving it and taking part in the risky venture. Write this value in the field below the figure 100% on the right side (100% means that you would receive this amount for sure).
b) If you feel that you would prefer to receive this sum than take part in the risky venture, then the value you have written is too high.
c) If you feel that you would prefer to take part in the risky venture than receive the sum, then the value you have written is too low.
d) Repeat steps a), b), c) until you are indifferent as to whether you take part in the risky venture or receive the sure sum of money.

Further comments:
Carefully consider the amounts given in the problems, and remember that you stand to gain real money. In fact, some of you will be selected to take part in a real risky venture after the experiment is finished.
Note that payoffs vary across problems.
Try to state the sure sum of money as precisely as possible – at least to within 5–20 zł. Avoid giving rounded amounts. The more precise your answers, the greater their academic worth.
Do not try to be “mathematically correct”. Obviously, you are not prohibited from counting. It might even be advisable that you do so. Keep in mind, however, that this is a psychological, and not a mathematical, test.
Before you complete the experiment, try one more example.
Example 2
…………….
If you understand the instructions, start the test by clicking “Next”.
If you are not sure about anything, read the instructions again.
If you do not wish to complete the test, press “Return”.

7.4. Appendix 4: Instruction used in Experiment 2

Instructions for cash values of gambles:
In this task, you are asked to judge the cash equivalent values of gambles. Each gamble can be thought of as a container holding several tickets. Each ticket has a prize value printed on it. You get to reach in the container and draw out one ticket blindly and at random, and the value printed on the ticket is your prize.
For example, consider the following case of a container holding exactly 4 tickets:
($30, $40, $50, $95) Cash Value = [$]
Each ticket is equally likely, so you might win $30, $40, $50, or $95. Would you like to get one of these prizes? Yes, you would. How much is the opportunity to reach in and draw out a ticket worth? We are not asking you to judge what you would pay for the opportunity to draw a ticket and win a prize, but it might be helpful for you to think of that amount as a starting place. Instead, you are asked to state an amount of money such that you would like the (sure) cash and the gamble equally well. To help you make a judgment, you might write down an amount in the box provided below, and then ask yourself, which you would rather have. Would you prefer the certain cash, or would you rather try the gamble? If you prefer the cash, then the amount you wrote down is too big. If you prefer the gamble, then the amount you wrote down was too small. If you like each option equally well, then your answer is just right. You should feel equally attracted to the money and the gamble.
Now, if you wrote something less than $30, then you would rather take the gamble, because the LEAST you could win with the gamble is $30, and you might win as much as $95. But the most you can win in any of these choices is $95, so your answer will be less than that. At first, you may find

the task a bit difficult, but you will soon be able to judge the cash values of the gambles. The first few trials are for practice. If you are not sure what to do, re-read the instructions, and if still unsure, raise your hand and ask for help.

You will notice that the number of tickets in the urn varies from two to ten. When there is only one ticket, or if all tickets have the same prize, you are guaranteed to win that amount, so its cash value is the same as the value printed on the ticket. For example, if it says $30, then you would be indifferent between taking the money ($30) and drawing $30 from the container. When there are two tickets, each ticket has a fifty-fifty chance. For example, ($30, $100) represents a fifty-fifty chance of winning either $30 or $100. The worst you could do with that gamble is $30 and the best you could do is $100.

($5, $95) Cash Value = $

Type a number in the box above and then ask yourself which you would rather have: the amount you typed or the gamble. If they are not equal, you should adjust the amount in the box so that they are equally good. Remember, the worst you could do with the gamble is $5, so your judgment should be greater than $5, and the most you could win is $95, so your judgment should be less than $95.

Please re-read the instructions to make sure you understand the task. When you understand the instructions, write in the amount of cash equal to each gamble below:

W1. ($5, $95) Cash Value = $

W2. ....

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