

Science of JDM as an Efficient Game of Mastermind

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Mastermind Game- Basic Game



Auf Deutsch => “SuperHirn”



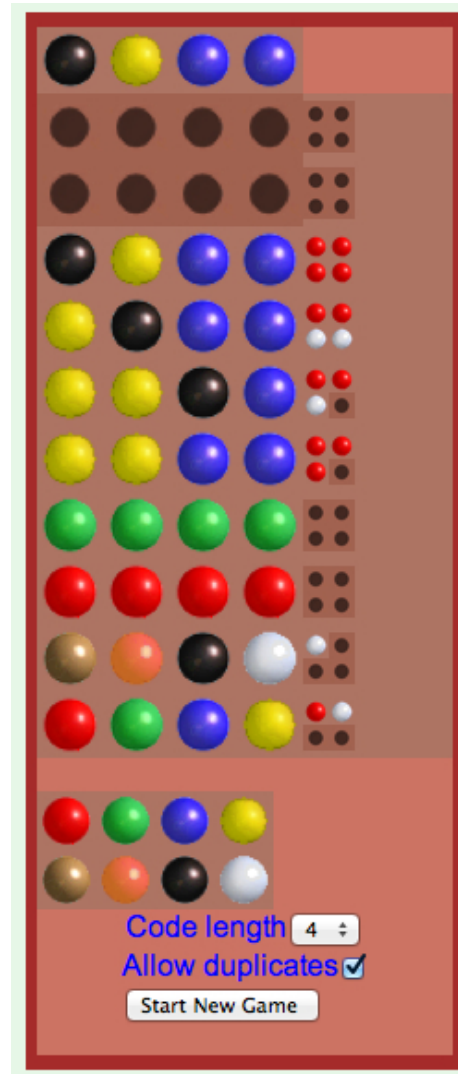
Mastermind Game

- Goal to find secret code of colors in positions. In “basic” game, there are 4 positions and 6 colors, so, $6^4 = 1296$ hypotheses. 8 colors: $8^4 = 4096$
- Each “play” of the game is an experiment that yields feedback related to the accuracy of an hypothesis.
- For each “play”, feedback = 1 colored peg for each color in correct position and 1 white peg for each correct color in wrong position.

Play Mastermind Online

- <http://www.web-games-online.com/mastermind/index.php>
- (Mastermind is a variant of “Bulls and Cows”, an earlier code-finding game.)

A Game of Mastermind- $4,096 = 8^4$



Analogies

- EXPERIMENTS yield results, from which we revise our theories.
- RECORD of experiments and results is preserved.
- Experiments REDUCE THE SPACE of theories compatible with evidence.
- Hypotheses can be PARTITIONED with respect to components.

Science vs. Mastermind

- In Mastermind, feedback is 100% accurate; in science, feedback contains “error” and “bias.” Repeat/revise the “same” experiment, different results.
- In Mastermind, we can specify the space of hypotheses exactly, but in science, the set of theories under contention expands as people construct new theories.
- In Mastermind, we know when we are done; science is never done.

Analogy

- EFFICIENT Mastermind is the goal: Find the secret code with fewest experiments.
- If FEEDBACK IS NOT PERFECT, results are fallible, and it would be a mistake to build theory on such fallible results.
- REPLICATION is needed, despite the seeming loss of efficiency.

Hypothesis Testing vs. Mastermind

- Suppose we simply tested hypotheses, one at a time and a significance tests says “reject” or “retain”?
- With 1296 hypotheses, we get closer to truth with each rejection--BARELY.
- Now suppose that 50% of the time we fail to reject false theories and 5% of the time we reject a true theory. Very slow progress.
- Clearly, significance testing this way is not efficient. More INFORMATIVE FEEDBACK needed.

Experiments that Divide the Space of Hypotheses in Half

- Basic game = 1296 Hypotheses
- Suppose each experiment cuts space in half:
1296, 648, 324, 162, 81, 40.5, 20.25, 10.1, 5.1, 2.5, 1.3, done. 11 moves.
- But typical game with 1296 ends after 4 or 5 moves, infrequently 6.
- So, Mastermind is more efficient than “halving” of the space.

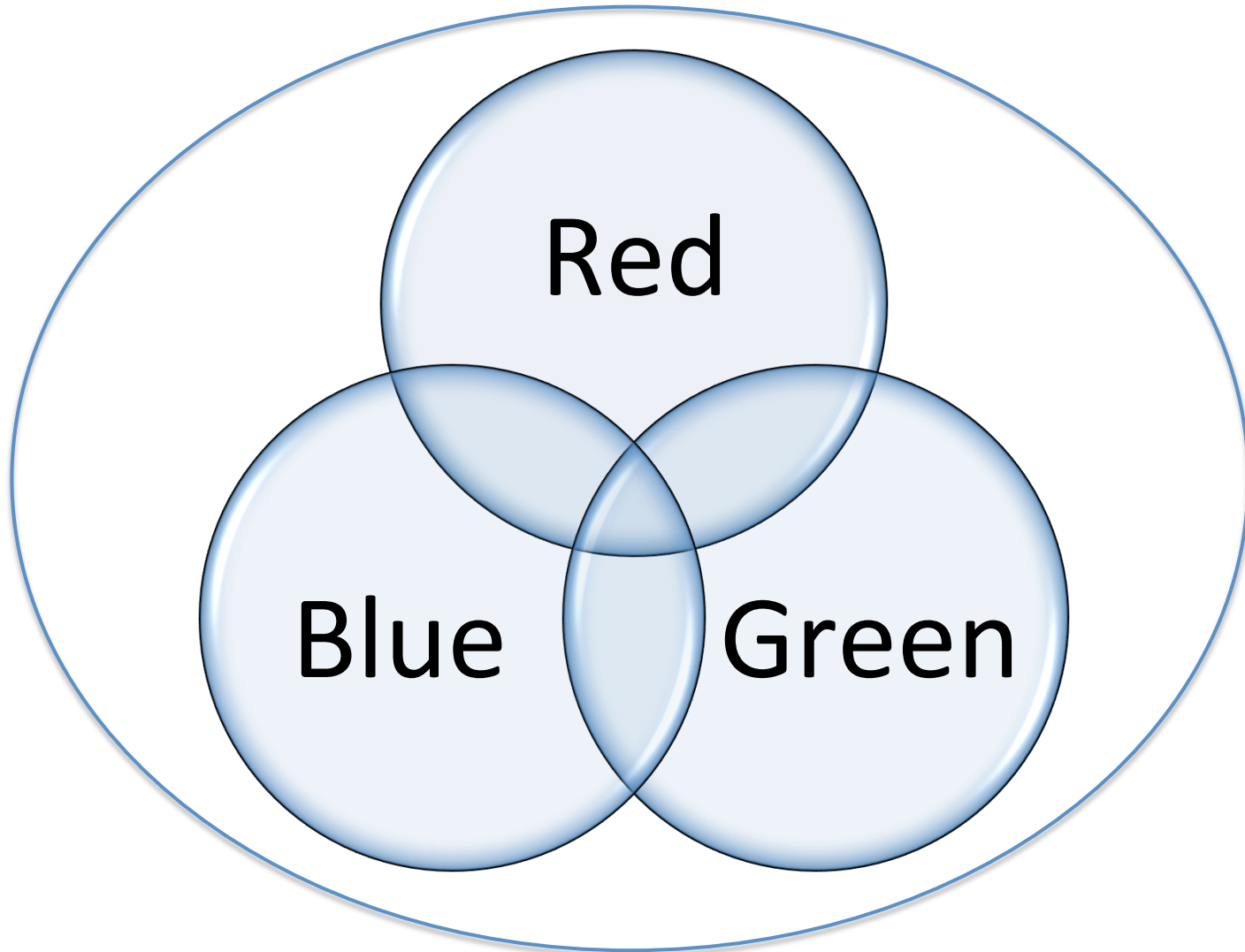
Index of Fit Informative?

- Suppose we assign numbers to each color, $R = 1$, $G = 2$, $B = 3$, etc. and calculate a correlation coefficient between the code and the experimental results?
- This index could be highly misleading, it depends on the coding and experiment.
- Fit can be higher for “worse” theories. (“Devil rides again” papers PB, 1970s).

Psychology vs. Mastermind

- Mastermind: only ONE secret code.
- In Psychology, we allow that different people might have different individual difference parameters.
- Even more complicated: Perhaps different people have different models.
- As if, different experiments in the game have DIFFERENT secret codes.

Partitions of Hypotheses



Testing Critical Properties

- Test properties that do not depend on parameters.
- Such properties partition the space of hypotheses, like the test of all REDs.
- For example: CPT (including EU) implies STOCHASTIC DOMINANCE. This follows for any set of personal parameters (any utility/value function and any prob. weighting function).

Critical Tests are Theorems of One Model that are Violated by Another Model

- This approach has advantages over tests or comparisons of fit.
- It is not the same as “axiom testing.”
- Use model-fitting to rival model to predict where to find violations of theorems deduced from model tested.

Summary

- Study of Alternative Explanations is like a game of Mastermind.
- Experiments designed to inform us as to the space of possible theories that are compatible with the data.
- Test critical properties to distinguish theories.

30 Years Later- Old Bull Story

