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### *Decision Under Risk*

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#### **Introduction**

Most decisions, whether choices to purchase flood insurance, invest overseas, pursue an experimental medical treatment, or steal a base, involve risk. Purchasing insurance is sensible if you believe a flood will happen, but a bad idea if you are convinced it won't. The study of risky decision making has addressed two broad questions. How *should* individuals behave when faced with a risky choice like the ones above? How *do* individuals behave when faced with a risky choice? The first question is *normative*; the second, *descriptive*.<sup>1</sup> Although the first question is clearly important, our aim in this chapter is to provide answers to the second question.

The study of risky decision making has a long, distinguished, and interdisciplinary history. The list of contributors include some of the most prominent figures in economics and psychology, including several Nobel Prize winners in Economics, and these ideas have in turn been applied with great success to business, law, medicine, political science, and public policy. We hope to give the reader an overview of the exciting developments made by these researchers and others. In particular, the goals of this chapter are fourfold:

- 1 survey the evolution of questions asked by researchers of risky decision making;
- 2 review the major intellectual contributions;
- 3 summarize the present state of knowledge; and
- 4 offer a research agenda for the next generation of research in the field.

We first distinguish between *risk* and *uncertainty* (Knight, 1921). Risk defines decision situations in which the probabilities are *objective* or given, such as betting on a flip of a fair coin, a roll of a balanced die, or a spin of a roulette wheel. Uncertainty defines

situations in which the probabilities are *subjective* (i.e., the decision maker must estimate or infer the probabilities), like the decision to invest overseas and the other examples given above. Although most important decisions clearly involve uncertainty rather than risk, we focus primarily on risk in this chapter. We do so, first, because risk is the simpler case and because there is considerably more empirical evidence on risk than uncertainty. But more importantly, we argue that our understanding of the simpler situation of risk readily extends to the more realistic case of uncertainty. In the latter sections of this review, we discuss how research on risk helps us understand decisions under uncertainty.

Before we begin, we point to the many excellent reviews of this sort that have been written over the years, (e.g., Camerer, 1995; Edwards, 1954, 1961; Fox & See, 2003; Luce, 2000; Machina, 1987; Mellers, Schwartz, & Cooke, 1998; Schoemaker, 1982; Starmer, 2000). We encourage those interested in the field to read these reviews; they provide a perspective of how the field has evolved over the years, and also highlight the differences and similarities between how economists and psychologists have approached this field.

## Expected Utility

We begin by reviewing the classical model of decision under risk, *expected utility* (EU) theory. Consider a gamble that gives  $p_i$  chance at  $x_i$ , which we represent  $(p_1, x_1; \dots; p_n, x_n)$ . The expected utility of this gamble is  $\sum p_i u(x_i)$ , where  $u(x_i)$  captures the “utility” of receiving outcome  $x_i$ . In expected utility, the burden of explaining risk attitudes falls completely on the shape of the utility function. Risk-averse behavior, such as the purchase of insurance, requires that the utility function be concave, while risk-seeking behavior, such as buying a lottery ticket, is explained by convexity of the utility function. Thus, it is difficult for EU to explain why an individual simultaneously purchases insurance and lottery tickets. This individual’s utility function must be concave for some wealth levels and convex for other wealth levels (Friedman & Savage, 1948). Nevertheless, economics usually assumes that decision makers are risk averse, the primary justification being diminishing marginal utility: a dollar to a pauper is considerably more useful than a dollar to Bill Gates (e.g., Varian, 1992).

Bernoulli (1738) proposed expected utility in the eighteenth century as a resolution of the famous St. Petersburg Paradox. The St. Petersburg gamble is a prospect that offers a  $1/2^n$  chance at  $\$2^n$  for  $n = 1, \dots, \infty$ . Although this gamble has an infinite expected value, most people would pay less than \$10 for this gamble. Many concave utility functions, including logarithmic and power utility functions, impose finite bounds on the maximum an individual would pay for the St. Petersburg bet. The contribution of Bernoulli, however, went far beyond reconciling this example. Bernoulli rejected expected value as a criterion for making risky choices, arguing more generally that two people with different desires and different wealth levels should not necessarily value the identical gamble equally. Although it is unclear whether Bernoulli was making a descriptive argument or normative argument, the generalization of expected value to expected utility was introduced and has remained important to this day.

Expected utility took off in the 1940s and 1950s when von Neumann and Morgenstern axiomatized the model in their *Games and Economic Behavior* (von Neumann & Morgenstern, 1947). Whereas Bernoulli assumed the EU representation, von Neumann and Morgenstern provided an axiomatic system: a set of conditions that were necessary and sufficient for expected utility. Axioms have a descriptive as well as normative benefit: they decompose a complex theory into smaller pieces, each of which can be tested empirically or scrutinized as normative principles. The most important axiom became known as the *Independence Axiom* or *Substitution Axiom*, and was reformulated by Marschak (1950) and Samuelson (1952). The basic idea of the axiom is straightforward. If you like gamble  $A$  more than gamble  $B$ , then you should prefer the mixture of  $A$  and some other gamble  $C$  (in some probabilistic proportion) to the mixture of  $B$  and  $C$  (in the same probabilistic proportion). The Independence Axiom can be stated formally: if  $A \succ B$  then  $pA + (1 - p)C \succ pB + (1 - p)C$ , where “ $\succ$ ” stands for the binary relation “is preferred to” and  $pA + (1 - p)C$  denotes a probabilistic mixture of  $A$  and  $C$ . To illustrate, suppose you prefer a .50 chance at \$100 ( $A$ ) to a .80 chance at \$50 ( $B$ ). The independence axiom requires that you prefer a .25 chance at \$100 to a .40 chance at \$50, since these gambles are derived by mixing the antecedent gambles with \$0 for sure ( $C$ ) in equal proportions.

In the early 1950s, there was considerable debate about the normative status of EU and the independence axiom and how to interpret EU’s utility function (Ellsberg, 1954; Friedman & Savage, 1952). When this debate finished, it was widely believed that EU was a compelling normative model (Savage, 1954). Indeed, in its abstract form, the independence axiom is intuitively compelling. Given a choice between  $pA + (1 - p)C$  and  $pB + (1 - p)C$ , the decision maker receives  $A$  or  $B$  if an unfair coin comes up heads, and  $C$  if the unfair coin comes up tails. If the coin comes up tails, it doesn’t matter what you chose. If the coin comes up heads, you should choose  $A$  if you like  $A$  more than  $B$ , and  $B$  otherwise. Thus, this logic argues that choosing between  $A$  and  $B$  is the same as choosing between  $pA + (1 - p)C$  and  $pB + (1 - p)C$ .

## Subjective Expected Utility

In 1954, Savage published the influential *Foundations of Statistics*. The major contribution was an axiomatic system that extended expected utility from risk to uncertainty. In uncertain situations, probabilities are not given, and outcomes depend on which event obtains. Consider a prospect,  $(E_1, x_1; \dots; E_n, x_n)$ , that offers  $x_i$  if event  $E_i$  occurs. The *subjective expected utility* (SEU) of this prospect is given by  $\sum \rho(E_i)u(x_i)$ , where  $u(\cdot)$  is a utility function as in standard expected utility and  $\rho(\cdot)$  is a *subjective probability measure* that obeys the standard axioms of probabilities. Thus, SEU is the natural generalization of EU from risk to uncertainty.

The critical axiom is Savage’s “Sure Thing Principle.” The sure thing principle shares the same basic intuition as the Independence Axiom, which we illustrate with the following example. Consider a choice between  $A$  and  $B$  in Table 20.1, where the outcome of the prospects depends on what event is realized:

**Table 20.1**

	$E_1$	$E_2$	$E_3$
$A$	100	0	0
$B$	0	100	0

A preference for  $A$  over  $B$  can be interpreted as a belief that  $E_1$  is more likely to occur than  $E_2$ . Since  $E_3$  shares a common outcome, 0, this event is irrelevant for the choice between  $A$  and  $B$ . Thus, a decision maker should have the same preferences if, as in Table 20.2, we substitute 50 (or any other outcome) for 0 in  $E_3$ :

**Table 20.2**

	$E_1$	$E_2$	$E_3$
$A'$	100	0	50
$B'$	0	100	50

Despite the normative appeal of the independence axiom and the sure thing principle, EU and SEU would soon be challenged as reasonable descriptive models of decision under risk and uncertainty.

## The Pre-prospect Theory Era

### *Edwards' review paper*

Are the elegant normative frameworks of Savage and von Neumann and Morgenstern descriptively accurate models? In a remarkable review of the decision making literature written only seven years after von Neumann and Morgenstern axiomatized EU, Edwards (1954) summarized the empirical and theoretical literature. There was already mounting empirical evidence that the normative theory of expected utility was descriptively inadequate. One pressure point came from the field, a need to explain the simultaneous purchase of insurance and lottery tickets. There were some impressive attempts to salvage the expected utility framework so that it could conform to this observation. For example, in a famous paper, Friedman and Savage (1948) attempted to account for the simultaneous purchase of insurance and gambling by introducing a utility function that had regions of convexity and regions of concavity (the former accounting for the risk seeking behavior of gambling and the latter accounting for the risk averse behavior of insurance purchase).

Another set of attacks came from the laboratory. Preston and Baratta (1948) found preliminary evidence that people distort given probabilities: small probabilities are overweighted and large probabilities are underweighted. This study, as well as other early empirical endeavors (e.g., Mosteller & Nogee, 1951; Davidson, Suppes, & Siegel, 1957) had various methodological problems, making it difficult to make inferences about the underlying mechanisms. These experiments were the best of the lot, leading Edwards (1954, p. 403) to remark of the others: "Such experiments are too seldom adequately controlled, and are almost never used as a basis for larger-scale, well-designed experiments."

Looking back, it is surprising how much was already known both empirically and theoretically by 1954. It is also surprising how far away researchers and theoreticians were from a comprehensive model of decision making under risk and uncertainty. Many of the major empirical results that characterized research in the 1980s were already known, but the lack of a proper theoretical framework kept researchers from fully understanding these results. As a child may misunderstand the clues given to her about where the last Easter egg is hidden, the researchers at the time were misled by many of the empirical results that were available – not merely because of the methodological deficiencies, but also because of the limited types of inferences the existing theoretical models permitted.

Edwards identified the fundamental problem of decision making research, "[the] development of a satisfactory scale of utility of money and of subjective probability" (p. 403). Indeed, Edwards also anticipated the theoretical problem that would characterize much research in the last 15 years: the composition rule that combines utility with distorted probabilities. In Edwards' words, "it seems very difficult to design an experiment to discover that law of combination" (p. 400).

### *The Allais and Ellsberg Paradoxes*

Allais (1953) posed the first major direct challenge to expected utility. He collected data at a Paris Colloquium in 1952 that was attended by a number of distinguished researchers interested in the foundations of decision theory. One of the problems Allais presented involved two choice pairs. The first pair was a choice between (*A*) \$1 million for sure, and (*B*) a .10 chance at \$5 million and .89 chance at \$1 million. (For simplicity, the .01 chance at \$0 with *B* is implicit.) The second pair involved a choice between (*C*) a .11 chance at \$1 million, and (*D*) a .10 chance at \$5 million. The modal choices of *A* and *D* are inconsistent with EU. This example became famously known as the *Allais Paradox*.

The intuition behind these two choices is the following. In the first choice, the sure thing of \$1 million is highly attractive, thus it is not worth risking a chance of nothing for the possibility of winning \$5 million. In the second choice, the two probabilities (.10 and .11) appear indistinguishable relative to the difference between \$5 million and \$1 million (e.g., Slovic & Tversky, 1974). Choosing *A* and *D* violates the independence condition. Note that *A* and *B* share a "common consequence" (a .89 chance at winning \$1 million), while *C* and *D* share a different common consequence (a .89 chance at winning \$0). Expected utility requires that a change in a common consequence not alter preference. So, if a decision maker chooses *A* over *B*, she should choose *C* over *D*. It is interesting that no careful empirical evidence for this choice pattern was collected

for over 10 years. MacCrimmon (1965) collected data for his PhD thesis (see also MacCrimmon, 1968), and replications were conducted by others (e.g., Morrison, 1967; Slovic & Tversky, 1974).

There was an analogous violation of the sure thing principle in the domain of uncertainty. Ellsberg (1961) proposed a famous problem that became known as the *Ellsberg Paradox*. Ellsberg did not publish data in that paper (although data appeared in his Harvard PhD thesis), but the violation was soon empirically verified by Becker and Brownson (1964) and then many others (see the review by Camerer & Weber, 1992).

Imagine an urn with 90 balls, 30 of which are red, and 60 of which are black or yellow in an unknown proportion (i.e., perhaps 0 black and 60 yellow, 60 black and 0 yellow, or any combination in between). Prospects link payments to whether a red, black, or yellow ball is drawn. Option *A* pays \$100 for red (and \$0 for black or yellow), while *B* pays \$100 for black (and \$0 for red or yellow). Now consider a variation on the above problem where the winnings for the yellow ball in both options are converted to \$100 instead of \$0. You win \$100 with *C* on red or yellow, and \$100 with *D* on black or yellow.

The predominant choice in the first pair is *A*, while the modal choice in the second pair is *D*. In both cases, subjects prefer betting on known probabilities to unknown or vague probabilities (*A* offers \$100 with a known chance of 1/3, while the chance of \$100 with *B* may be anywhere between 0 and 2/3). However, these choices are incompatible with the sure thing principle. As Ellsberg described the pattern, “The first pattern, for example, implies that the subject prefers to bet ‘on’ red rather than ‘on’ black; and he also prefers to bet ‘against’ red rather than ‘against’ black” (p. 654).

## Prospect Theory

Kahneman and Tversky’s (1979) “Prospect theory: An analysis of decision under risk” is the second most cited papers in economics during the period, 1975–2000 (Coupé, in press; Laibson & Zeckhauser, 1998).<sup>2</sup> The paper’s success is probably due to its unique combination of simplicity and depth. Kahneman and Tversky presented convincing empirical demonstrations that highlighted some general descriptive deficiencies with expected utility, as well as a powerful formal theory for organizing these demonstrations. Although the Allais Paradox was now 25 years old, very little data existed challenging expected utility, and there were no theoretical alternatives to the classic model.

The classic model of decision under risk assumes that individuals are generally risk averse (perhaps because of diminishing marginal utility). However, Kahneman and Tversky demonstrated that people are risk-averse *and* risk-seeking and that the pattern of risk attitudes can be organized in a remarkably simple manner. They found that 84 percent of subjects preferred \$500 for sure to a .50 chance at \$1,000, but 72 percent preferred a .001 chance at \$5,000 to \$5 for sure. The first choice demonstrates risk aversion for moderate probabilities, the second risk-seeking for small probabilities. When choices involve losses, the pattern reversed: 69 percent chose a .50 chance at losing \$1,000 to losing \$500 for sure, and 83 percent chose losing \$5 for sure over a .001 chance at losing

Table 20.3

	<i>Small probabilities</i>	<i>Medium to large probabilities</i>
Gains	Risk-seeking	Risk-averse
Losses	Risk-averse	Risk-seeking

\$5,000. For losses, subjects were risk-seeking for moderate probabilities and risk-averse for small probabilities.

These data are typical of a more general pattern, called the *reflection effect*: preferences tend to reverse when the sign of the outcomes is changed. This pattern has become known as the *four-fold pattern of risk attitudes* and can be summarized as in Table 20.3 (Tversky & Kahneman, 1992).

Kahneman and Tversky also presented two direct violations of expected utility, the common-consequence effect and common-ratio effect. Expected utility can explain the four-fold pattern by positing a specific utility function, but doing so would strain the theory (Rabin, 2000). The common-consequence and common-ratio effects are examples of a different approach: a direct test of one of the axioms underlying expected utility. A violation of any of the EU axioms falsifies the EU model – there can be no utility function that can accommodate the pattern.

We begin with the *common consequence effect*, an example that follows the same basic schema as Allais' original demonstration. Most subjects preferred \$2,400 for sure to a .33 chance \$2,500, a .66 chance at \$2,400, and a .01 chance at \$0, but preferred a .33 chance at \$2,500 to a .34 chance to \$2,400. To show that no utility function can reconcile this pattern, it suffices to observe that the first choice under EU reduces to  $.34u(2,400) > .33u(2,500)$ , while the second choice simplifies to  $.34u(2,400) < .33u(2,500)$ . Indeed, this example was constructed to test the independence axiom directly. One pair is derived from the other pair by substituting a .66 chance at \$2,400 for a .66 chance at \$0 (hence the name common consequence effect).

Why do the majority of subjects violate expected utility? The reasoning may go something like this: "In the first choice, it is not worth sacrificing a sure thing for a chance at getting a slightly better outcome. In the second choice, neither gamble is a sure thing. Since .33 and .34 are very similar, it is worth going for the better outcome, \$2,500." This pattern demonstrates the *certainty effect*: subjects are willing to pay a large premium to avoid a small chance of receiving nothing.

In the *common-ratio effect*, subjects chose \$3,000 for sure to a .80 chance at \$4,000, but a .20 chance at \$4,000 to a .25 chance at \$3,000. This pattern also contradicts expected utility, since the first choice implies  $u(3,000) > .8u(4,000)$ , but the second implies  $.25u(3,000) < .2u(4,000)$ . The independence axiom is violated in this example as well since the second pair is constructed by mixing a 25 percent chance of the first pair with a 75 percent chance of receiving \$0. The effect gets its name because the ratio of the probability of winning \$4,000 to the probability of winning \$3,000 is the same for both choices.

Kahneman and Tversky proposed a formal theory to explain the four-fold pattern, common-consequence effect, common-ratio effect, and a slew of other demonstrations. The major components were a *value function* and a *probability weighting function*. In expected utility theory, the utility function is defined over final wealth states, an assumption that is known as asset integration. In contrast, the value function in prospect theory,  $v(\cdot)$ , is defined over changes in wealth rather than absolute wealth levels. The function is concave for gains, convex for losses, and exhibits “loss aversion”, i.e., the function is steeper for losses than gains (see Chapter 19, this volume).

The probability weighting function,  $\pi(\cdot)$ , captures how different probability levels contribute to the evaluation of a gamble. Tversky and Kahneman presented a schematic weighting function that overweights small probabilities, and underweights medium and large probabilities. They also suggested that there might be a discontinuity at the end points of 0 and 1: differences between 0 and 1 either would be ignored, or there would be a categorical distinction between 0 (“impossibility”) and some small probability (“possibility”) and a distinction between 1 (“certainty”) and some large probability close to 1 (“uncertainty”). The weighting function also exhibits *subcertainty*,  $\pi(p) + \pi(1 - p) < 1$ . Roughly speaking, the weighting function is more below the identity line (for moderate and high probabilities) than it is above the identity line (for low probabilities).

Prospect theory combines the two functions as follows. Consider a gamble  $(p, x; q, y; 1 - p - q, 0)$ , where  $p + q < 1$  and  $x$  and  $y$  may both be positive, both negative, or one positive and one negative. The value of such a gamble is given by  $U(p, x; q, y) = \pi(p)v(x) + \pi(q)v(y)$ . This functional form explains the four-fold pattern of risk attitudes, common-consequence effect, and common-ratio effect in terms of the qualitative properties on  $v(\cdot)$  and  $\pi(\cdot)$  discussed above. This form also has the same basic structure as expected value and expected utility. Expected utility is the sum of utility values weighted by probabilities. Prospect theory generalized this notion by summing utility values weighted by transformed probabilities or *decision weights*.

Recall that EU had a hard time reconciling simultaneous purchasing of insurance and gambling because the utility function had to do all the heavy lifting. The burden of explaining risk attitudes now falls on the value function and the probability weighting function. In prospect theory, insurance purchasing and gambling are explained by the overweighting of small probabilities. Insurance purchasing is risk-averse behavior: thus overweighting of small probabilities has to be large enough to overcome the convexity of the value function in losses. Similarly, gambling represents risk-seeking, which is predicted if overweighting of small probabilities is sufficient to overcome the concavity of the value function in gains.

We return to Edwards’ (1954) question about how to separate distortions of value from distortions of probabilities. The prospect theory representation permits independent inferences about the value and weighting function. For example, restrictions on the weighting function can be inferred from the common-consequence effect problems, and restrictions on the value function (i.e., concavity and loss aversion) can be inferred from other examples presented in their paper.<sup>3</sup>

It is a surprise to many people that many of the ideas from prospect theory existed in previous literatures. In a remarkable paper, Markowitz (1952, p. 154) proposed a utility

function that defined utility as changes from present wealth as an attempt to capture the observation that individuals at just about every wealth were insurance buying gamblers. Preston and Baratta (1948) investigated choices involving varying probability levels and found a pattern remarkably similar to that captured by prospect theory's weighting function (see also Mosteller & Nogee, 1951; Davidson et al., 1957). Edwards (1953) documented probability preferences, preferences to bet on certain probabilities when faced with bets of equal expected value, and argued that descriptive models needed to take account of the nonlinear impact probabilities have on decisions. Finally, MacCrimmon (1968) and MacCrimmon and Larsson (1979) presented similar demonstrations of the common-ratio and common-consequence violations, and Williams (1966) documented rejection of fair gambles, consistent with loss aversion (see, also, Mosteller & Nogee, 1951; Slovic & Lichtenstein, 1968, p.10).

Thus, many of the pieces of prospect theory, taken alone, were not novel. However, the reputation of prospect theory as one of the most important papers in social science is nevertheless completely deserved. The paper took ideas that had been around, some for as long as 30 years, scattered in different literatures and thought to be unrelated, and constructed a formal model in which all the elements worked together. The paper also produced compelling demonstrations (some extensions of old findings and some new predictions such as the isolation and pseudo-certainty effects, and the rejection of probabilistic insurance). Even those with no interest in formal theory could nevertheless understand why expected utility was an unsuitable descriptive model and what was required of an adequate descriptive model. Most importantly, the view that the Allais Paradox was an isolated problem for expected utility was no longer tenable.

## The Post-prospect Theory Era

### *An overview*

Kahneman and Tversky attacked the independence axiom and expected utility in an elegant, coherent, and convincing manner. Since the Allais Paradox could no longer be considered an isolated anomaly, prospect theory forced economists to consider EU violations seriously. In this section, we review the quarter-century of research following prospect theory. The phase can best be understood as an ongoing dialogue between alternative models to EU and ingenious tests conducted to discriminate among these alternative models.

In hindsight, the post-prospect theory models appear to be motivated by two different concerns, those of economists and those of psychologists. In general, economists strove for a descriptive theory of decision under risk that was elegant, general, and mathematically tractable. This strategy was practical, as much as aesthetic. Expected utility had been applied with great success to many important areas of economics, such as game theory and information economics (e.g., Pratt, 1964; Rothschild & Stiglitz, 1970). The new models relaxed the independence axiom and hence are "generalizations" of EU in the

sense that EU is a special case of these models. The common-consequence and common-ratio effect violations presented in the last section posed a minimum standard for a set of new models.

Why did theorists need an alternative to prospect theory when prospect theory explained the basic violations just fine? There are at least three reasons. Many theorists disliked the prospect theory representation since it admitted possible violations of stochastic dominance:  $(p, x; q, x - \epsilon)$  could exceed  $(p + q, x)$ , even though the second gamble stochastically dominates the first gamble (Fishburn, 1978).<sup>4</sup> To avoid this problem, Kahneman and Tversky proposed an editing operation where subjects spotted dominated alternatives. Second, theorists wanted to have a model that included EU as a special case. Finally, prospect theory was limited to two non-zero outcomes.

In contrast, psychologists were generally more concerned with explaining the underlying psychological process. Some alternative models were cognitive, while others considered personality and motivational factors (e.g., Birnbaum & Stegner's (1979) configural weight theory and Lopes' (1987) aspiration-level theory). These models tended to have more free parameters than prospect theory, and therefore were more flexible but less tractable and parsimonious.

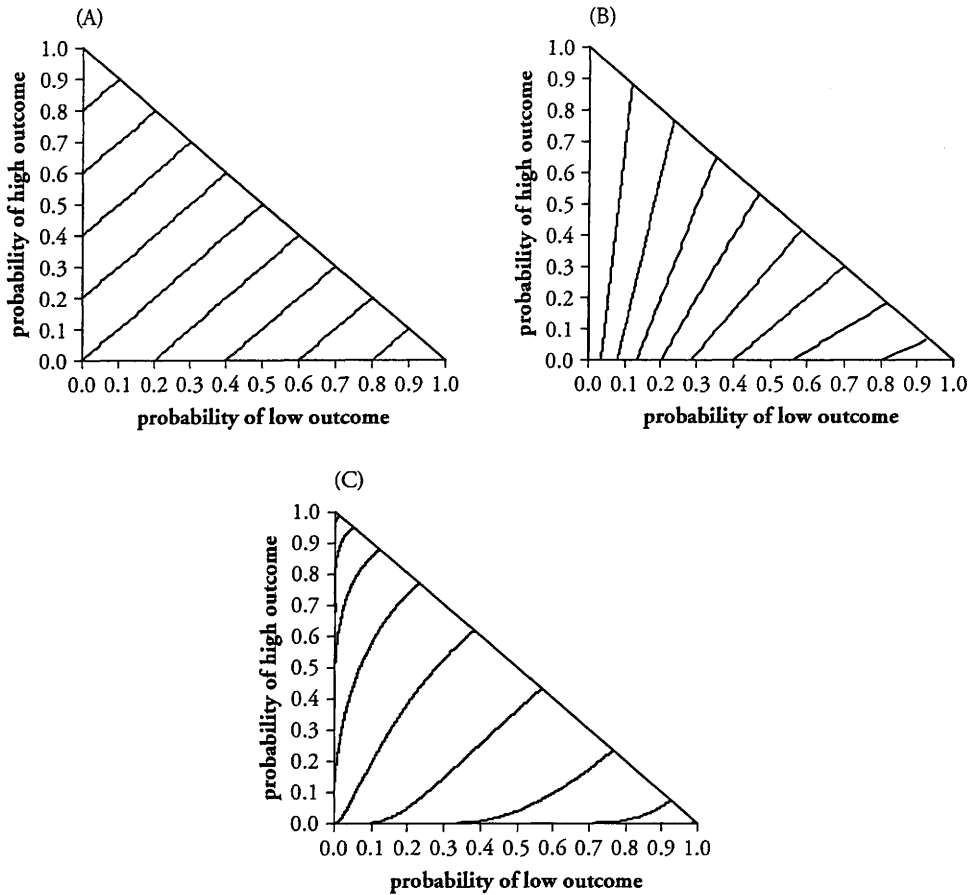
At the end of these 25 years of research, prospect theory stands out as the best descriptive model. However, prospect theory, too, evolved as part of this dialogue. One important fruit of this stream of theoretical and empirical work was a refinement of prospect theory, cumulative prospect theory (CPT; Tversky & Kahneman, 1992). Although original prospect theory (OPT) was groundbreaking, it had clear limitations, some of which were acknowledged in the original paper (for a historical perspective, see Kahneman, 2000). Not only did cumulative prospect theory overcome the potential violations of dominance that some saw as a flaw of OPT, CPT generalized prospect theory to apply to an arbitrary number of outcomes, and uncertainty as well as risk. We discuss details of CPT later.

One simple device greatly facilitated the dialogue, the probability triangle or simplex, used originally by Marschak (1950) and later adopted by Machina (1982). The unit triangle provided a common "language" and platform for understanding the implications of different models and visualizing and organizing empirical findings. Models could be understood in terms of restrictions placed on the curvature, slope, and fanning property of the indifference curves in the triangle.<sup>5</sup> Figure 20.1 illustrates indifference curves for some of the major theories.

### *Alternatives to prospect theory*

Since the common-consequence and common-ratio effects violate the independence axiom, an adequate descriptive model of decision under risk needs to "relax" the independence axiom in some respect. Many generalized EU models took the strategy of replacing the independence axiom with a weaker form. We discuss several different families of model that were advanced in the 1980s.

Machina (1982) took a non-axiomatic approach. EU requires that indifference curves be linear and parallel. Machina suggested an ingenious hypothesis called the *fanning-out*



**Figure 20.1** The probability triangle is used to depict gambles with at most three outcomes

The  $x$  axis captures the probability of the lowest outcome, the  $y$  axis the probability of the highest outcome. Shown are indifference curves for (A) expected utility theory; (B) weighted utility with indifference curves that “fan out”; (C) rank-dependent utility. All gambles on an indifference curve yield the same utility.

*hypothesis*. Indifference curves no longer had to be linear or parallel, but just needed to “fan-out”. Fanning-out requires that decision makers became more risk-averse as lotteries improve in the sense of first-order stochastic dominance. Graphically, fanning out hypothesis posits that the indifference curves in the unit triangle become steeper when moved in the northwest direction (see Figure 20.1b). Fanning-out of indifference curves is consistent with the basic common-consequence and common-ratio effects.

One family of models replaced the independence axiom with a weaker form called *betweenness*. Betweenness requires that a probabilistic mixture of two lotteries should be in the middle of the two lotteries in preference, i.e., for any two lotteries  $A$  and  $B$ : if  $A \geq B$  then  $A \geq pA + (1 - p)B \geq B$ . Betweenness is intuitively appealing as well as pragmatic: some important economic applications only require betweenness and not the full force of the independence axiom (Crawford, 1990).

A number of betweenness models were proposed, including weighted utility theory (Chew & MacCrimmon, 1979; Chew, 1983), implicit weighted utility theory (Dekel, 1986), skew-symmetric bilinear utility theory (Fishburn, 1984), Neilson's (1992) boundary effect hypothesis, and disappointment-aversion theory (Gul, 1991). Graphically, betweenness requires that indifference curves in the probability triangle be straight lines (as with EU), but not necessarily parallel (as required by EU) (see Figures 20.1a and 20.1b). With appropriate parameters, these models can accommodate the basic common-consequence and common-ratio effects. These models mainly differ in the fanning properties of the indifference curves, either imposing uniform fanning (all fanning in or all fanning out) or mixed fanning.

Rank-dependent models are variants of prospect theory. In prospect theory, there is a nonlinear transformation of outcomes, as well as probabilities. Original prospect theory permits violations of dominance or monotonicity, a problem that Kahneman and Tversky recognized and dealt with in the editing phase (but see Tversky & Kahneman, 1986). *Rank-dependent* utility (RDU) was an ingenious way of allowing probability distortions, like prospect theory, while prohibiting violations of dominance. The basic idea is to transform *cumulative* probabilities instead of individual probabilities (Quiggin, 1982; see also, Luce, 1988; Yaari, 1987). A prospect,  $(p, x; q, y)$ , where  $x > y$ , would be valued by  $\pi(p)v(x) + [\pi(p+q) - \pi(p)]v(y)$ . The *decision weight*, the amount that a particular outcome is weighted, depends on the probability of that outcome as well as the rank of that outcome in the gamble. More generally, the value of a prospect,  $(p_1, x_1; \dots; p_n, x_n)$ , where  $x_i > x_{i+1}$ , is given by:

$$\sum_{i=1}^n \left( \pi \left( \sum_{j=1}^i p_j \right) - \pi \left( \sum_{j=1}^{i-1} p_j \right) \right) v(x_i).$$

In this general form, the decision weight for an outcome  $x_i$  is the probability weighting function applied to the probability of receiving at least outcome  $x_i$  minus the weighting function applied to the probability of receiving at least outcome  $x_{i-1}$  (for a useful introduction, see Diecidue & Wakker, 2001).<sup>6,7</sup>

Regret theory took a different approach to generalizing EU. One carrier of value is *regret*, the comparison between the outcome received and the outcome that would have been received under some other choice. Bell (1982) and Loomes and Sugden (1982) independently demonstrated how particular forms of regret theory could explain a wide range of phenomena, including purchasing of insurance and lotteries, the reflection effect, the Allais paradox, probabilistic insurance, and preference reversals. (We do not review tests of regret theory here, however see Starmer, 2000.)

### *Critical empirical evidence*

A slew of tests were designed to discriminate between the various models. The most discerning tests either tested general axioms such as betweenness or general features of preferences such as the fanning-out hypothesis. This approach was efficient: choice patterns inconsistent with these axioms or features ruled out a whole family of models.

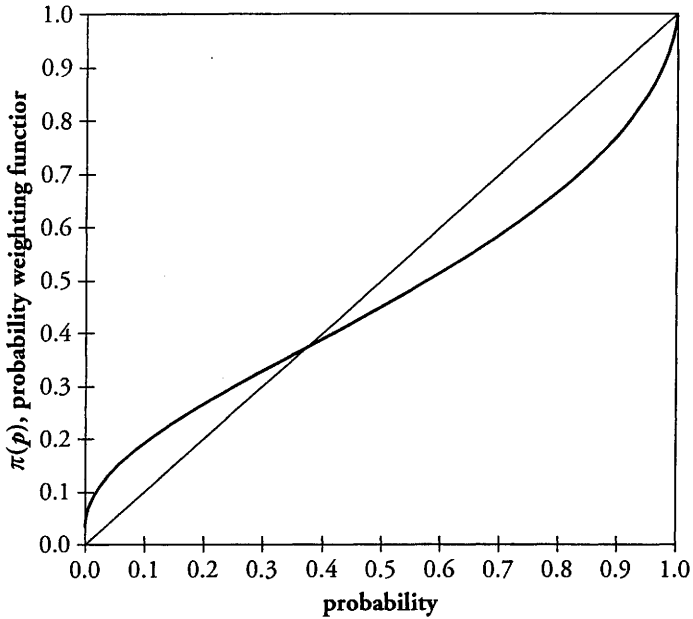
Machina's (1982) hypothesis that indifference curves fan-out everywhere in the unit triangle stimulated many empirical tests. Conlisk (1989) generalized the Allais Paradox and the common-consequence effect. He found that subjects preferred (.10,\$5M; .89,\$1M) over (.20,\$5M; .78,\$1M), but preferred (.98,\$5M) over (.88,\$5M; .11,\$1M). This pattern violates fanning-out, since preferences become more risk-seeking as gambles are improved. This particular problem generalizes the common-consequence effect in the following sense. In Allais' example and Kahneman and Tversky's (1979) original demonstration, probability mass is shifted from the lowest to the middle outcome, which corresponds to a horizontal movement of gamble pairs in the unit triangle. Here, the shift is from the middle to the highest outcome, corresponding to vertical movement of gambles pairs. Similar results showing that indifference curves that fan-in vertically are found in a variety of studies (Battalio, Kagel, & Jiranyakul, 1990; Camerer, 1989; Starmer & Sudgen, 1989; Wu & Gonzalez, 1998). Prelec (1990) demonstrated fanning-in in a very different part of the triangle. His subjects preferred (.02,\$20,000) to (.01, \$30,000) but (.01,\$30,000; .32,\$20,000) to (.34,\$20,000). Wu and Gonzalez (1996) found similar patterns of fanning-in along the bottom edge.

Empirical evidence showing mixed-fanning within the unit triangle ruled out models assuming uniform fanning-out or uniform fanning-in, such as weighted utility theory, implicit weighted utility theory, and Machina's (1982) fanning-out hypothesis. Other models allowed mixed-fanning, such as Gul's disappointment aversion theory (1991), Neilson's (1992) boundary effect hypothesis, rank-dependent utility, and prospect theory.

Another set of studies investigated betweenness. If betweenness is violated, all betweenness models are falsified. Prelec (1990) documented a stunning violation of betweenness. He found that 94 percent of subjects preferred  $A = (.17, \$20,000)$  to  $C = (.17, \$30,000)$  but 82 percent of subjects preferred  $B = (.01, \$30,000; .32, \$20,000)$  to  $A = (.34, \$20,000)$ . Since  $B = \frac{1}{17}C + \frac{16}{17}A$ , betweenness requires that  $B$  should lie between  $A$  and  $C$  in preference. Other empirical tests with gambles located in the southeast corner found similar patterns (Battalio et al., 1990; Camerer, 1989, 1992; Camerer & Ho, 1994). Prelec offered a very intuitive way of interpreting his finding. People may find trading a 2 percent chance of \$20,000 for a 1 percent chance of \$30,000 attractive, thus choose  $B$  over  $A$ . However, they do not like taking 17 such trades, which means exchanging a 34 percent chance of \$20,000 for a 17 percent chance of \$30,000.

The direction of violations is also useful for distinguishing between models. Violation of betweenness could be due to either quasi-concave preferences (i.e., convexity of indifference curves) or quasi-convex preferences (i.e., concavity of indifference curves), indicating a preference for or against randomization, respectively (Camerer, 1992). Prelec's example constitutes quasi-concave preferences. Quasi-convex preferences have been found for gambles located in the northwest corner (Battalio et al., 1990; Camerer, 1989, 1992; Camerer & Ho, 1994), while both quasi-concavity (Chew & Waller, 1986; Gigliotti & Sopher, 1993) and quasi-convexity (Conlisk, 1989) are found in the southwest corner of the triangle. Finally, betweenness violations tend to be weaker for gambles located inside the unit triangle than for gambles on the boundary.

Are any of the models described above consistent with the pattern of mixed fanning and quasi-concave and quasi-convex preferences? An appropriate model needs to be "nonlinear in probability" in order to capture betweenness violations. It turns out that



**Figure 20.2** A characteristic probability weighting function

betweenness data are consistent with both prospect theory and rank-dependent utility models, assuming an inverse S-shaped weighting function (see Camerer & Ho, 1994). Consider the weighting function depicted in Figure 20.2. The weighting function is concave for small probabilities and convex for medium and large probabilities. This shape generates indifference curves that are convex in the southeast corner, concave in the north corner, and mixed in the south corner (Camerer & Ho, 1994), consistent with the general findings. It also predicts the fanning-in patterns found by Prelec (1990) and Conlisk (1989), located in very different regions (Wu & Gonzalez, 1998). Finally, these models capture the diminished EU violations in the interior of the triangle (see Camerer, 1992).

### *Refinements of prospect theory*

In 1992, Tversky and Kahneman proposed cumulative prospect theory. The new prospect theory used the same basic building blocks as original prospect theory: a value function, defined over gains and losses, and a weighting function that captured probability distortions. The major technical innovation was to use the rank-dependent form to extend prospect theory to an arbitrary number of outcomes and to uncertainty as well as risk. For risk, a separate rank-dependent transformation was applied to the gain and loss portions of a prospect. The weighting function for gains and losses is also possibly sign-dependent. For uncertainty, CPT used a related model that had been developed for uncertainty, Choquet Expected Utility (Gilboa, 1987; Schmeidler, 1989).

The new theory unified the basic shape of the value and weighting function according to one psychophysical principle. Consider the probability weighting function depicted in Figure 20.2. This particular form of the weighting function explains the betweenness violations as well as the fanning patterns discussed in the previous section. For outcomes, concavity of gains and convexity of losses reflects diminishing sensitivity away from the reference point of 0. Concavity of the weighting function for small probabilities and convexity of the weighting function for large probabilities reflects the same principle applied to different reference points: diminishing sensitivity away the boundaries of 0 (impossibility) and 1 (certainty). Consider three individuals endowed with a 0 chance to win 100, a .33 chance to win 100, and a .99 chance to win 100. How might these three individuals view a .01 chance improvement in the chance of winning? An inverse S-shaped weighting function suggests that individuals are most sensitive to changes near the extremes and relatively insensitive to changes in the middle. Thus, the individuals endowed with a 0 chance to win, and a .99 chance to win will view the change much more favorably than the person with a .33 chance to win. In weighting function terms,  $\pi(.01) - \pi(0) > \pi(.34) - \pi(.33)$ , and  $\pi(1) - \pi(.99) > \pi(.34) - \pi(.33)$ .

Tversky and Kahneman (1992) presented a comprehensive empirical test of the model. Subjects provided cash equivalents for a number of gambles, differing in the probability and magnitude of the highest outcome, and involving gains and losses. The vast majority of subjects exhibited the four-fold pattern of risk attitudes: risk aversion for most gains and low probability losses, and risk seeking for most losses and low probability gains. A parametric regression analysis of the cash equivalents produced an S-shaped value function, and a weighting function of the form of Figure 20.2. Many other studies, using a variety of methodologies, find a similar inverse S-shape (e.g., Abdellaoui, 2000; Bleichrodt & Pinto, 2000; Camerer & Ho, 1994; Tversky & Fox, 1995; Wu & Gonzalez, 1996, 1998). The weighting function is found to intersect the identity line somewhere between  $.30 < p < .40$ . It is noteworthy that Kahneman and Tversky's original common-consequence effect demonstration used values that approximately maximize the size of the EU violation.

Wu and Gonzalez used a common-consequence schema to produce a non-parametric trace of the weighting function. They started with a choice between  $(p, x)$  and  $(q, y)$ , where  $x > y$ . Both gambles were improved in increments by the common consequence  $(r, y)$ , with  $r$  increasing throughout the probability range. Consistent with an inverse S-shaped weighting function, they found the percentage of subjects choosing the risky gamble increases and then decreases. For example, 38 percent of subjects preferred  $(.05, \$240)$  to  $(.07, \$200)$ , 65 percent preferred  $(.05, \$240; .30, \$200)$  to  $(.37, \$200)$ , and 39 percent preferred  $(.05, \$240; .90, \$200)$  to  $(.97, \$200)$ .

There have been numerous efforts to parameterize the weighting function. Tversky and Kahneman (1992) assumed a one-parameter form of the weighting function,

$$\pi(p) = \frac{p^\gamma}{((p^\gamma + (1-p)^\gamma)^{1/\gamma})}, \text{ and fit this form to cash equivalent data. The fitted form}$$

had the characteristic inverse S-shape, with a crossover point of around  $p = .39$ . Other exercises have produced similar parameter estimates, even though these exercises varied considerably in terms of the data used (choice versus cash equivalent) estimation techniques

(nonlinear regression of cash equivalents versus fitting stochastic choice functionals) (see Abdellaoui, 2000; Camerer & Ho, 1994; Tversky & Fox, 1995; Wu & Gonzalez, 1996).

More recently, Gonzalez and Wu (1999) estimated a two-parameter weighting function to median data and individual subject data,  $\pi(p) = \frac{\delta p^\gamma}{\delta p^\gamma + (1-p)^\gamma}$ . The estimated function for median data resembled previous analyses, but there was considerable heterogeneity at the level of individual subjects. This weighting function has two parameters to capture the relative flatness or steepness of the weighting function (“curvature” through  $\gamma$ ) and the relative level (“elevation” through  $\delta$ ) of the weighting function. Adding a second parameter did not appreciably improve the fit for the median data, but both parameters were needed to model the heterogeneity of individual subjects. Although all 10 subjects had inverse S-shaped weighting functions, there was considerable variation along the two dimensions. While the majority of subjects exhibited sub-certainty ( $\pi(p) + \pi(1-p) < 1$ ), some subjects exhibited super-certainty ( $\pi(p) + \pi(1-p) > 1$ ). Some subjects had weighting functions that were close to the identity line, while others had weighting functions that approximated step functions.

The two-parameter weighting function has also proved useful for understanding how literally the reflection effect holds. Tversky and Kahneman (1992) estimated weighting and value functions for gains and losses. The value function had the same coefficient for both gains and losses,  $v(x) = x^{.88}$ . The weighting function had nearly identical coefficients for gains and losses,  $\gamma = .61$  for gains and  $\gamma = .69$  for losses, leading Tversky and Kahneman to conclude that: “the weighting functions for gains and losses are quite close, although the former is slightly more curved than the latter.” In a different study using a very different elicitation methodology, Abdellaoui (2000) estimated nearly identical parameters,  $\gamma = .60$  for gains and  $\gamma = .70$  for losses. However, Abdellaoui also fitted the two-parameter function used by Gonzalez and Wu (1999) and found significant differences between losses and gains: the weighting function was significantly more elevated for losses ( $\delta = .84$ ) than gains ( $\delta = .65$ ), whereas the curvature parameters were nearly identical. Similar results have been found by Abdellaoui, Vossman, and Weber (2002). Indeed, a re-analysis of Tversky and Kahneman’s (1992) data found the same sign-dependence in elevation. We used Tversky and Kahneman’s (1992) median data (Table 20.3) and assumed the two-parameter weighting function and  $v(x) = x^{.88}$  for gains and losses. A nonlinear regression produced estimates of  $\delta = .79$  and  $\gamma = .60$  for losses and  $\delta = .88$  and  $\gamma = .67$  for gains.

### *Extension to uncertainty*

The Allais and Ellsberg Paradoxes led researchers to treat decision under risk and decision under uncertainty differently for many years. More recent evidence suggests a unification of these results. Many of the principles underlying decision under risk apply directly to decision under uncertainty. For example, Tversky and Fox (1995) tested two conditions, lower- and upper-subadditivity conditions, for both risk and uncertainty. Roughly, these conditions can be thought of as capturing the possibility and certainty effects. They found strong support for lower- and upper-subadditivity in both domains, but more

subadditivity for uncertainty than risk. The main reason that individuals were less sensitive to uncertainty than risk is that probability judgments were subadditive as well, consistent with support theory (Tversky & Koehler, 1994). This led Tversky and Fox (1995) (see also Fox & Tversky, 1998) to suggest a two-stage model, in which an uncertain prospect  $(E, x)$  is valued by  $W(E)v(x) = \pi(p(E))v(x)$ , where  $\pi(\cdot)$  is the probability weighting function for risk, and  $p(E)$  is the subjective probability of  $E$ .

Tversky and Kahneman (1992) generalized the common consequence effect from risk to uncertainty and found qualitatively identical results. More recently, Wu and Gonzalez (1999) extended concavity and convexity conditions from risk to uncertainty and found a nearly identical U-shaped pattern of risk preferences as Wu and Gonzalez (1996). Thus, it seems that the same general principles apply to risk and uncertainty. Below, however, we discuss some ways that risk and uncertainty differ.

## Future Research

In the past ten years, a relatively clear picture of risky decision making and prospect theory has emerged. Violations of expected utility are robust and systematic, and prospect theory seems to explain these violations with the most ease. Basic properties of the value and weighting function, qualitatively as well as quantitatively, can organize these violations, and these basic principles readily extend from risk to uncertainty. Parsimonious parametric forms of prospect theory fit choice data well, at both the aggregate level and at the level of individual subjects.

However, the picture is somewhat incomplete, as some parts of prospect theory have received very little empirical attention. We highlight two of these “loose threads” and suggest some avenues for future research in these areas.

### *Simplification and evaluation*

Kahneman and Tversky (1979) proposed an editing phase, which was designed “to organize and reformulate the options so as to simplify subsequent evaluation and choice” (p. 274). They proposed six operations: coding, combination, segregation, cancellation, simplification, and detection of transparent dominance. Discussion of editing does not appear until the conclusion of Tversky and Kahneman’s (1992) revision of prospect theory, and it is tempting to conclude that rank-dependent representation obviates the need for editing operations. However, Tversky and Kahneman conclude with an apt quote: “Theories of choice are at best approximate and incomplete . . . When faced with a complex problem, people employ a variety of heuristic procedures in order to simplify the representation and evaluation of prospects” (p. 317). Below, we discuss some open issues in the processes of simplification and evaluation.

Almost no research exists on how decision makers code and represent gambles. Research on managerial decision making has found that managers tend to focus on the best and

worst outcomes (the “upside” and “downside”) with almost no attention to the probability of the outcomes (March and Shapira, 1987). Beyond this, while decision makers almost certainly use operations to simplify gambles, it is not well-understood when individuals use within-gamble operations such as combination, or across-gamble operations such as cancellation. Some theories that evoke similarity as an across-gambles operation exist (e.g., Leland, 1998; Rubinstein, 1988), but a more comprehensive theory of editing is out-of-sight at the moment. We suspect that this sort of theorizing is probably the wrong strategy. Instead, we hope that researchers document violations of EU that can be plausibly explained by some editing operation (Wu, 1994). When an ample set of findings have been assembled, it might then be possible to build a more comprehensive theory of editing.

In terms of evaluation of gambles, more research needs to be conducted in two areas: mixed gambles and composition rules. The little research on mixed gambles is particularly surprising since most real world gambles involve some possibility of gain and some possibility of loss, at least relative from the status quo. Prospect theory and other bilinear models have problems explaining some of the mixed gamble data collected to date (Chechile & Butler, 2000). We hope to see more data of this sort collected, particularly data collected to test axioms that permit separability between gains and losses, such as Tversky and Kahneman’s (1992) “double matching.”

Finally, original and cumulative prospect theory differ in terms of how the weighting and value functions are combined, i.e., the composition rule used. The overall evidence is mixed about which prospect theory is better in explaining the empirical results. The two prospect theories are identical for prospects with one non-zero outcome, and thus provide the same explanation for the common-ratio effect. The models however diverge for more complicated gambles. Both models can explain the original common-consequence violations, as well as generalizations of the common-consequence violations (e.g., Wu & Gonzalez, 1996, 1998), albeit with different restrictions on the weighting function. Beyond that, there are some patterns that cannot be explained by CPT (Wu, 1994), and some patterns that CPT fits better than OPT (Fennema & Wakker, 1997; Wakker, 2003). In goodness of fit tests using particularly parametric forms of the value and weighting function, the pattern is mixed. OPT sometimes fits aggregate data better (Camerer & Ho, 1994; Wu & Gonzalez, 1996), but CPT fits particular patterns better (Wu & Gonzalez, 1996). Birnbaum and McIntosh (1996) and Birnbaum, Patton, and Lott (1999) present choice patterns that cannot be explained by CPT, although some can be explained by OPT. Gonzalez and Wu (2003) used parameters estimated from two-outcome gambles, where CPT and OPT coincide, to predict three-outcome gambles, where the models diverge. Neither model did particularly well predicting the cash equivalents from the three-outcome gamble holdout sample. OPT tended to overpredict, while CPT tended to underpredict three-outcome gamble cash equivalents. Finally, Wu, Zhang, and Abdellaoui (2004) adapt Abdelloui’s (2002) tradeoff consistency conditions to create a critical test of the two prospect theories. They find that CPT is violated for critical test gambles that do not involve a sure thing, while OPT is violated for gambles that involve a sure thing.

Which composition rule is better is an open question. The mixed results suggest that there may be no general answer. The answer probably ultimately depends on a number

of factors, including the components of the choice set (Stewart, Chater, Stott, & Reimers, 2003), and whether editing rules can be applied. Gonzalez and Wu (2003) offer some suggestions about how composition rules may be related to areas of cognition, including attention, information processing, and similarity.

### *Source preference*

How does the evaluation of a gamble that pays \$100 with .7 chance differ from the evaluation of a prospect that offers \$100 if the Yankees win tomorrow's game? The difference between these gambles captures the difference between risk and uncertainty. The research reviewed above suggests that these two choices are qualitatively similar. However, we suggest that there are some differences. Of course, uncertainty is more complicated than risk: decision makers faced with the sports bet must assess the likelihood of a Yankee victory. Tversky and Fox's (1995) two-stage model suggests that individuals judge the probability of a particular event and then transform this judgment via a probability weighting function. Under this simple form, a decision maker who judges the likelihood of a Yankee win to be .7 will value the risky gamble and the uncertain gamble the same.

Although the two-stage model is a useful simplification, it fails systematically in some situations. Decision makers may prefer to bet on one source over another, even when the subjective likelihood is equated for the two sources. The clearest example is the Ellsberg Paradox (Ellsberg, 1961). In the two-urn problem, most subjects assign a probability of .5 to both of the two urns, yet nevertheless prefer betting on the objective .5 to the subjective .5 (see Camerer & Weber, 1992, for a review).

Numerous empirical studies have demonstrated some sort of source preference. Heath and Tversky (1991) found that individuals prefer to bet on domains in which they felt particularly competent to domains in which they felt less competent, even when subjective probabilities for the two domains were matched. Fox and Tversky (1995) found that the Ellsberg Paradox was reduced or disappeared in between-subject tests. Evidently, subjects are not averse to ambiguity per se, but only when they feel comparatively ignorant (see also, Frisch & Baron, 1988). More recently, Kilka and Weber (2001) measured the degree of source preference directly. German students valued prospects based on a familiar source, the price of Deutsche Bank, a German bank, and an unfamiliar source, the price of Dai-Icho Kangyo Bank, a Japanese bank. The weighting function for the familiar source was significantly more elevated than the weighting function for the more unfamiliar source.

The few studies to date suggest that source dependence acts on the elevation of the weighting function, rather than on the curvature. Illusion of control (Langer, 1975) can be seen as working through elevation of the weighting function. We suspect, however, that there may be effects that work through curvature of the weighting function as well. A decision maker who does not feel particularly knowledgeable about a source, such as politics, may judge one event to be more likely than another, but may attach the same value to a gamble based on the first event compared to a gamble based on the second event. Thus, we hypothesize a flatter weighting function for sources in which subjects feel comparably ignorant.

Source preference complicates matters in one other respect. Decision theorists have assumed that beliefs can be inferred from actions (Ramsey, 1931). For example, suppose that you are indifferent between a risky bet, \$100 with a .7 chance, and an uncertain event, \$100 if the Yankees win tomorrow's game. Under the standard interpretation, indifference means that you judge the probability of the event in question to be .7. However, a decision-based definition of probability is elusive if decision makers prefer to bet on one source over another (Wakker, 2004).

## Final Thoughts

This chapter has taken the reader through a tour of the many phases in the history of risky decision making research. We also have tried to provide some preview of what the future holds in store. We close by suggesting two additional fertile research areas. First, we suspect that the study of decision under risk and uncertainty will increasingly consider the role of affect and emotion. Initial investigations in this area have proven quite promising (e.g., Mellers, Schwartz, Ho, & Ritov, 1997; Rottenstreich & Hsee, 2001; Chapter 22, this volume). Second, we see researchers in economics and finance importing these theories and models at an increasing rate. This trend includes research that uses these models to generate more general theoretical predictions (Barberis, Huang, & Santos, 2001), as well as work that uses functional specifications in more flexible structural models (cf. Barberis & Thaler's (2003) review of behavioral economics).

## Notes

- 1 A third category is *prescriptive*: how to get ordinary people to act more normatively? This particular question motivates decision analysis (e.g., Raiffa, 1968). Many decision analysts see the divergence between descriptive and normative models as an argument why decision analysis is needed (e.g., Bell, Raiffa, & Tversky, 1988).
- 2 Kahneman and Tversky originally envisioned a theory of regret, but "abandoned this approach because it did not elegantly accommodate the pattern of results that we labeled 'reflection' . . ." (Kahneman, 2000).
- 3 The common consequence effect demonstration reveals subcertainty of  $\pi(\cdot)$ , while rejection of an asymmetric fair gamble,  $(x, .50; -x, .50)$  reveals loss aversion,  $v(x) < -v(-x)$ , and a preference for  $(p, x; p, y)$  over  $(p, x + y)$  reveals concavity of  $v(\cdot)$ :  $v(x) + v(y) > v(x + y)$ .
- 4  $A$  stochastically dominates  $B$  if it has at least as high a probability of any outcome  $x$ , and a strictly better probability of some outcome  $y$ .
- 5 However, the unit triangle method is limited to lotteries having at most three outcomes. More complex lotteries therefore cannot be studied within the unit triangle paradigm. For more details on the probability triangle, see Camerer (1989) and Machina (1987). For reviews and details of non-expected utility models, see Camerer (1992) and Fishburn (1988).
- 6 We do not discuss the axioms underlying rank-dependent utility models, since they tend not to be transparent or easily tested. There are at least two exceptions, axiom systems based on ordinal independence (Green & Jullien, 1988) and tradeoff consistency (Abdellaoui, 2002).

- 7 Note that the rank-dependent form appeared in Kahneman and Tversky (1979) for two-outcome gambles,  $(p, x; 1 - p, y)$ , where  $x > y > 0$ .

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