

## Chapter 15: Psychological Scaling with “Ratios” and “Differences”

Does it make sense to describe how you feel? Can one measure psychological impressions such as those of heaviness of lifted weights, darkness of a patch of color, the loudness or pitch of a tone? If subjective values of psychophysical stimuli can be measured, there is hope that psychology can become a science in which it is meaningful to discuss subjective experiences for other concepts such as pain, pleasure, hunger, perceptions, emotions, and judgments. In the last chapter, you learned about the presentation of psychophysical stimuli, and in this chapter you will learn about the scaling of psychological values of stimuli that do not necessarily have physical measures.

Previous chapters analyzed single experiments. In this chapter, you will learn to analyze two different experiments that are linked in the same study. The concept that links them is the assumption that subjective experiences can be measured, and that these subjective values are independent of the comparison task.

If you have not done so already, participate as a judge in the two experiments on “ratios” and “differences” of prestige of occupations. Is the difference in prestige between the occupations of *doctor* and a *factory worker* greater than the difference in prestige between the occupations of *college professor* and *architect*? Most people think so. If such judgments can be represented by intervals on a numerical scale, it is possible to assign numbers to represent the separations in prestige between occupations. In other words, we can scale subjective values on a social dimension.

These experiments were constructed with the help of factorWiz. The row and column stimuli are names of occupations. The experiments are available on the CD from the list of examples. These experiments illustrate how such judgments can be used to test

models of comparison and also to measure subjective value. In this example, the studies scale the prestige of different occupations. Data are included on CD for two experiments that form a Web-based partial replication of the experiments by Hardin and Birnbaum (1990).

Thurstone (1927) argued for a subtractive model of comparison. However, Stevens (1957) argued that one should use “ratio” judgments and a ratio model. Taking judgments of “ratios” at face value, he thought category ratings and judgments of intervals were “biased and invalid.” Early research, comparing the method of magnitude estimation with the method of equal appearing intervals, such as described in Chapter 14 could not resolve these rival theories. The fact that these two procedures did not yield the same scale was known, but could not be resolved by single studies.

Modern research allows us to not only test these models, but also to test theories having implications for the results of two or more different types of experiments. Evidence with judgments of “ratios” and “differences” has been consistent with the theory that people use only one operation, that the operation is subtraction, and that psychophysical functions come closer to the predictions of Fechner than to those of Stevens (Birnbaum, 1980). This chapter will teach you how to analyze such research.

### **A. Subtractive Model**

Imagine three points on a straight line,  $A$ ,  $B$ , and  $C$ . The distance from  $A$  to  $C$  equals the distance from  $A$  to  $B$  plus the distance from  $B$  to  $C$ . If all of the differences between points, are known, one can not only assign numbers to the points (measure), but also test the model, which assumes these points are on a line.

In a test of the subtractive model, the experimenter presents pairs of stimuli such as squares varying in darkness, weights varying in heaviness or tones varying in loudness and asks people to judge the psychological “differences” between the stimuli. Quotation marks are used to distinguish the instruction to judge “differences” or numbers produced by people under such instructions from actual (computed) or theoretical differences. Judgments of “differences” or “ratios” may or may not obey the mathematical properties of numerical or theoretical differences or ratios. As you will read later in this chapter, evidence is consistent with the theory that people actually judge “ratios” by computing subjective differences (Birnbaum, 1980, 1982).

The subtractive model of “difference” judgments can be written as follows:

$$D_{ij} = a_D(s_i - t_j) + b_D \quad (15.1)$$

where  $D_{ij}$  is the predicted judgment of “difference” between two stimuli, having subjective values,  $s_i$  and  $t_j$ ;  $a_D$  and  $b_D$  are constants that depend on the response scale. This model implies that there should be no interaction between two factors in which the stimuli are varied. For any set of four stimuli, the following should hold,

$$D_{ij} - D_{im} = D_{kj} - D_{km} . \quad (15.2)$$

For example, the “difference” in prestige between *physician* and *factory worker* minus the “difference” between *physician* and *college professor* should equal the “difference” between *police officer* and *factory worker* minus the “difference” between *police officer* and *college professor*. Judgments in any two rows should be linearly related to each other, with the same slope. In other words, there should be no interaction between Rows and Columns in Analysis of Variance.

## B. Ratio Model

The ratio model of “ratio” judgments can be written as follows:

$$R_{ij} = a_R (s_i / t_j)^m + b_R \quad (15.3)$$

where  $R_{ij}$  is the predicted “ratio” of stimuli with the same scale values as in Equation 15.1, and where  $a_R$ ,  $b_R$ , and  $m$  are constants that depend on the response scale. When  $a_R$  and  $m$  are both 1, and  $b_R = 0$ , we say that judgments of “ratios” can be *taken at face value*, so if a person says that one job is “three times” as prestigious as another, we assume the subjective ratio is 3. If  $m = 2$ , for example, it means that if a person says the “ratio” is “four times”, we assume the subjective ratio is 2 (because  $2^m = 4$ ).

Equation 15.3 implies that judgments of “ratios” should form a divergent, bilinear fan in which any two rows are linearly related and all pairs of curves intersect at the same point on the ordinate. This pattern is like that of the multiplicative model examined in Chapter 13, restricted to positive numbers.

## C. Two-operation Theory

The two-operation theory assumes that when people are instructed to judge both “differences” and “ratios,” that Equations 15.1 and 15.3 describe both sets of judgments with the same scale values (same values of  $s$  and  $t$ ) in both tasks. Actual ratios and differences are not monotonically related to each other. For example,  $2 - 1 = 3 - 2$  but  $2/1 > 3/2$ . Similarly,  $2/1 = 4/2$ , but  $2 - 1 < 4 - 2$ . Therefore, if people used two operations, one predicts that judgments of “ratios” and “differences” will not be monotonically related to each other, but instead show the theoretical pattern of interrelations of actual ratios and differences.

## D. One-operation Theory

The one-operation theory assumes that people use the same scale values and the same comparison operation for both tasks (Torgenson, 1961). The subtractive theory of Birnbaum (1980; 1982) postulates that “ratio” judgments are governed by the following subtractive model instead of Equation 15.3:

$$R_{ij} = a_R \exp(s_i - t_j) + b_R, \quad (15.4)$$

where  $\exp(x)$  represents the exponential function. This theory predicts that “ratio” and “difference” judgments will be monotonically related to each other,  $R_{ij} = \mathfrak{N}(D_{ij})$ , where  $\mathfrak{N}$  is a strictly increasing monotonic function, because both are based on the same underlying operation computed on the same scales of subjective value. In other words, if “ratios” and “differences” are each monotonically related to the same intervals of subjective value, then they should be monotonically related to each other.

By collecting data for two experiments, “ratios” and “differences,” employing the same stimuli, it is possible to test between the one- and two-operation theories.

## E. Clean and Filter “Ratios” and “Differences” of Occupational Prestige

To analyze the data on CD, follow the steps described in previous chapters to filter the data, and copy them to a new workbook. In this case, the steps are as follows: From Excel, open the data file, *clean.xls*. Type variable names in the first row (if you have not done so already). Then click in cell A1, and choose *Filter: AutoFilters* from the **Data** menu. Click on the dropdown selection arrow in A1, and select *pres\_diffs*, from the list of experiments. Drag the mouse from the second row in Column A to Column AG, then continue to drag down to select all of the data (excluding the first row of variable names). Select *Copy* from the **Edit** menu (you should now see the “ants” crawling

around to show your selection). From the **File** menu, select *New*, to create a new workbook. Click in cell A2 of the new workbook, and choose *Paste* from the **Edit** menu (or press CTRL and V). You have now pasted the judged “differences.” Save the new workbook as *RD.xls* (select *Save As* from the **File** menu).

Now, return to *clean.xls* (from the **Window** menu, select the filename *clean.xls*.) Click again in Cell A1 on the dropdown selection, and now select *pres\_ratios*. As before, click in the second row of Column A and drag to the right to Column AG then drag down to select all of the rows of data (excluding the first row), choose *Copy* from the **Edit** menu. Then return to the file, *RD.xls*, by clicking on that filename in the **Window** menu. Now click the tab at the bottom of the sheet labeled *Sheet2*. Click in Cell A2, and choose *Paste* from the **Edit** menu. You now can double click on the tab and rename *Sheet2* to *R* (for judgments of “ratios”). Double click on the tab *Sheet1* and rename it *D* (for judgments of “differences”).

Now look at the last row of sheet *R* (“ratios”). That row contains a list of variable names. That list was created by the trick (described in Chapter 11) of completing the experiment by typing in the stimuli instead of responses; then in an editor replacing the date, time and gender with “date,” “time,” and “sex.” Select that row and cut it from the last row, then paste it into the first row of both worksheets (with “ratios” and “differences.”) You may need to delete a blank row. To delete a row, click on the row number (to the left of the grid), which will select the row. Then from the **Edit** menu, select *Delete* (but be careful not to delete the whole sheet!). Make the variable names bold by selecting the first row and clicking on the “B” for bold on the toolbar (or select *Cells* from the **Format** menu, and click the *Font* tab). Now copy them to sheet *D*

(“differences”) as follows: select the first row, then select *Copy* from the **Edit** menu, then click on the tab for *D* and paste in the first row. You should edit the first entry, now in Cell A1, to say *Differences*, instead of *pres\_ratios*.

The column labels for the judgments can be rotated to vertical as follows. Select Columns G through AF in the first row, select *Cells* from the **Format** menu, then click the tab for *Alignment*, and drag the red dot to turn the text sideways.

The data file appears as in Figure 15.1. If you examine the file, you will see that on 12/1/98, at 3:03PM a 30-year old female made a judgment of 3,333,333. Since the instructions specified responses from 1 to 9, this response is completely out of range. Perhaps she put her finger on the 3 and held the key long enough to produce a string of repeated 3s. As noted earlier, you should follow a preplanned scheme to put aside data of subjects who do not follow instructions for separate analysis. It would be misleading to average such out-of-bounds values with the rest of the data. When finding means, one error that large makes quite a difference in the result. In this case, the column mean would be 22,224.72 instead of 2.52.

Insert Figure 15.1 about here.

To find such out-of-range numbers, use conditional formatting. Select the data (In Row 2, select H through AF, then drag down to include all rows, aside from the first). When the data are selected, choose *Conditional formatting...* from the **Format** menu. The dialog box is shown in Figure 15.2. Specify that values that are not between 1 and 9 will have a yellow background and red font. Examining the file, you will find a subject who left six trials blank. That row of data should also be removed for separate analysis. From identifying information (not shown), it was found that person “A” repeated the

task, submitting two sets of data. That person's data are labeled. To avoid giving too much weight to data from those who submit multiple copies, you should follow a predetermined rule to use only one set of data per person.

Repeat the process with the judgments of "ratios," looking for numbers less than or equal to zero. You will discover a few typos. Change '00 to 100), and change 800 to 800—the letter "oh" was used instead of zero. These cases also be spotted because they are left-justified, like text, instead of right-justified, as are numbers in Excel.



Figure 15.1. Appearance of the data file, *RD.xls*. Check the value in Cell K17. Instructions were to use integers from 1 to 9; thus, 3,333,333 must be an error.

Microsoft Excel - RD.xls

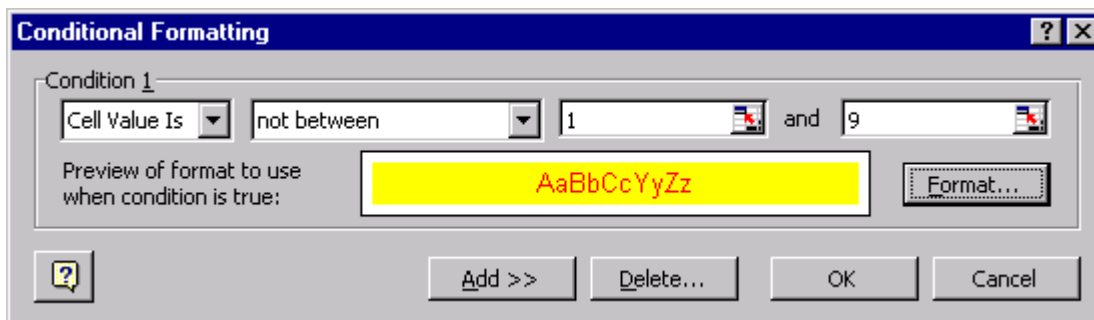
File Edit View Insert Format Tools Data Window Help

3333333

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R
1	"Differences"																	
2	pres_diffs	11/30/98	4:25:21 PM	20	M	12	American	1	1	1	1	1	2	2	1	2	1	
3	pres_diffs	11/30/98	5:31:19 PM	18	F	12	USA	2	3	2	2	3	4	4	5	2	2	
4	pres_diffs	11/30/98	6:11:37 PM	18	F	12	USA	4	5	2	2	1	4	4	5	2	2	
5	pres_diffs	11/30/98	9:05:41 PM	18	F	18	USA	5	4	4	1	1	5	5	5	2	2	
6	pres_diffs	11/30/98	9:28:06 PM	18	F	12	Filipino	5	4	4	3	3	5	5	5	3	3	
7	pres_diffs	12/1/98	9:00:16 AM	33	F	16	U.S.A.	5	5	5	2	2	6	5	5	3	2	
8	pres_diffs	12/1/98	11:39:41 AM	18	F	12	usa	5	5	4	3	2	5	5	5	2	2	
9	pres_diffs	12/1/98	1:25:50 PM	24	M	12	United Sta	3	5	4	3	3	4	4	5	3	3	
10	pres_diffs	12/1/98	1:47:34 PM	18	M	12	CHINESE	2	5	2	2	2	5	5	3	5	7	
11	pres_diffs	12/1/98	2:06:30 PM	18	M	13	USA	5	5	3	3	1	5	5	5	2	2	
12	pres_diffs	12/1/98	2:30:03 PM	18	F	12	TAIWAN	5	4	1	2	1	6	6	5	3	2	
13	pres_diffs	12/1/98	2:38:33 PM	22	F	15	usa	5	4	3	2	2	5	5	5	3	3	

Ready

Figure 15.2. The conditional formatting dialog box. In this case, the cell will have a yellow cell background and red font if the value is not between 1 and 9. The instructions for the “difference” task instructed people to use a 1 to 9 scale, so values outside of this range are “out of bounds.”



## F. Find Averages and Arrange Data Matrices

The next step is to find means. Click in Cell H111 (just after the end of the data) in Sheet R (“ratios”). Choose *Function* from the **Insert** menu, then select *AVERAGE* from the list of functions. Select H2:H110 for the range of numbers to be averaged (Excel will probably correctly anticipate this for you). Then hit enter. Next, use *AutoFill* to fill in the means for all of the columns by dragging the *AutoFill* handle (+) from H111 to AG111.

The next step is to arrange the Column means in an array. From the **Insert** menu, choose *Worksheet*. Name this new worksheet *matrices*. Now, copy the mean “ratio” judgments into Cell C3 of the new worksheet (use *Paste Special* and be sure to click *Paste Link*). Then cut the last 20 cells and paste them in Cell C4. Next cut off the last 15 cells and paste in Cell C5. Continue cutting and pasting groups of five in this way until you have a five by five matrix. Then type in the names of the occupations in the rows and columns. Next, find the row and column means of the matrix using the *AVERAGE* function, and using *AutoFill* to speed up the process. Copy the row marginal means into Column B using *Copy* and *Paste Special (Paste Link)*. After making the labels and marginal means bold, and after formatting the cells to show three decimals, the matrix appears as in Figure 15.3. Insert Figure 15.3 about here.

Find the means of the “difference” judgments using *AVERAGE*. *Copy* and *Paste Special (Paste Link)* these means to Cell L3 in the worksheet “matrices.” Cut and paste them to arrange them in a similar matrix of average (arithmetic mean) judgments. Figure 15.4 shows the matrix of mean “differences” after the marginal means have been computed. Insert Figure 15.4 about here.

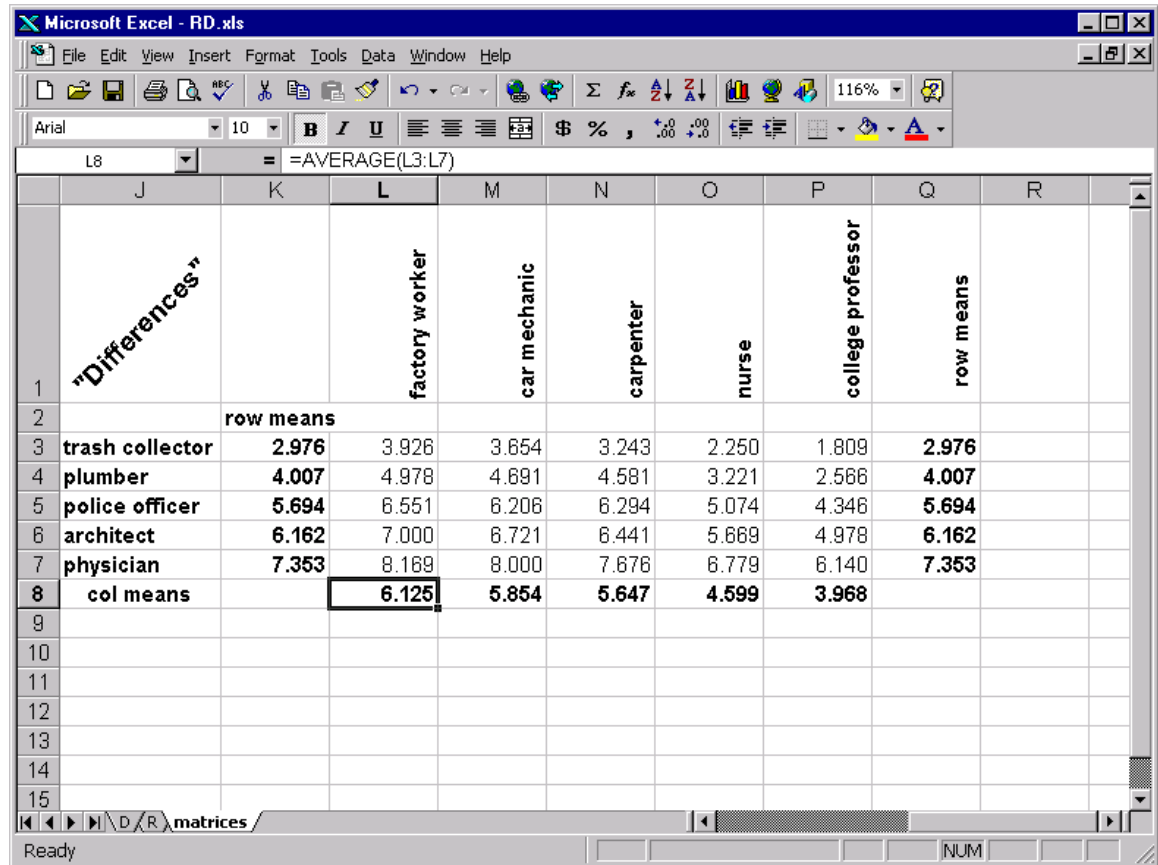
Figure 15.3. Appearance of the matrix of mean “ratios” of prestige of the row occupation divided by the column occupation. For example, it is 4.78 times as prestigious to be a *physician* than a *factory worker* (recall that judgments are 100 times the “ratios”), and it is 2.03 times as prestigious to be a *physician* as a *college professor*. The row means were copied from H3:H7 to B3:B7 to facilitate drawing the graph.

The screenshot shows the Microsoft Excel interface with the file 'RD.xls' open. The formula bar displays the formula  $H3 = \text{AVERAGE}(C3:G3)$ . The spreadsheet contains a matrix of data with row and column means calculated.

	A	B	C	D	E	F	G	H	I
1	"ratios"		factory worker	car mechanic	carpenter	nurse	college professor	row means	
2		row means							
3	trash collector	51.735	80.156	65.638	58.367	31.005	23.509	51.735	
4	plumber	78.523	113.106	100.000	93.450	47.463	38.596	78.523	
5	police officer	176.342	236.766	227.271	196.720	134.450	86.505	176.342	
6	architect	235.681	322.018	339.908	222.156	184.078	110.243	235.681	
7	physician	348.344	478.440	446.101	348.853	265.252	203.073	348.344	
8	col means		246.097	235.783	183.909	132.450	92.385		
9									
10									
11									
12									
13									
14									
15									

The status bar at the bottom indicates 'Ready' and 'NUM'.

Figure 15.4. Matrix of mean “differences” in occupational prestige. The scale ranged from 1 to 9, with 5 indicating “no difference” in prestige. The data show that it is very much more prestigious to be a *physician* than a *factory worker*. The entry in P3 is less than 5, showing that it is more prestigious to be a *college professor* than a *trash collector*.



### G. Make Graphs of “Ratio” and “Difference” Judgments

Graph the “ratio” judgments against the row marginal means. To do this, select B3:G7 and click the chart icon, or select *Chart* from the **Insert** menu. From the Chart Wizard, select *XY (Scatter)* and select *...points connected by straight lines*. In Step 2, click that data Series are in *Columns*. In Step 3, click *Titles* and add appropriate labels for the abscissa and ordinate, also click *Gridlines* and uncheck any gridlines. In Step 4, insert the graph as a *New sheet*. To adjust the appearance of the figure, point and double click on chart elements to modify them (you can also bring up menus with the right mouse button on PC or CTRL and click on Mac). After changing the colors of the points and lines, adjusting the font sizes of the labels, and formatting the numbers, the “ratio” data appear as in Figure 15.5. Insert Figure 15.5 about here.

Follow the same procedures to make a figure of the “difference” judgments, plotting them in the same way as the “ratio” judgments. The resulting graph is shown in Figure 15.6. Insert Figure 15.6 about here.

Notice that the “ratio” judgments are not parallel, but show divergence. The “difference” judgments are nearly parallel and linear. At first look, the data appear to conform to the theory that people compute ratios when instructed to judge “ratios” and that they compute differences when instructed to judge “differences.” That is, each data matrix appears consistent with the idea that people followed the directions for that task. However, another way of looking at the data contradicts that theory and suggests that they are using the same computation in both tasks.

Figure 15.5. Geometric mean judgments of “ratios” of occupational prestige as a function of the row marginal means, with a separate curve for each “denominator” occupation.

The successive numerator occupations were *trash collector*, *plumber*, *police officer*, *architect*, and *physician*.

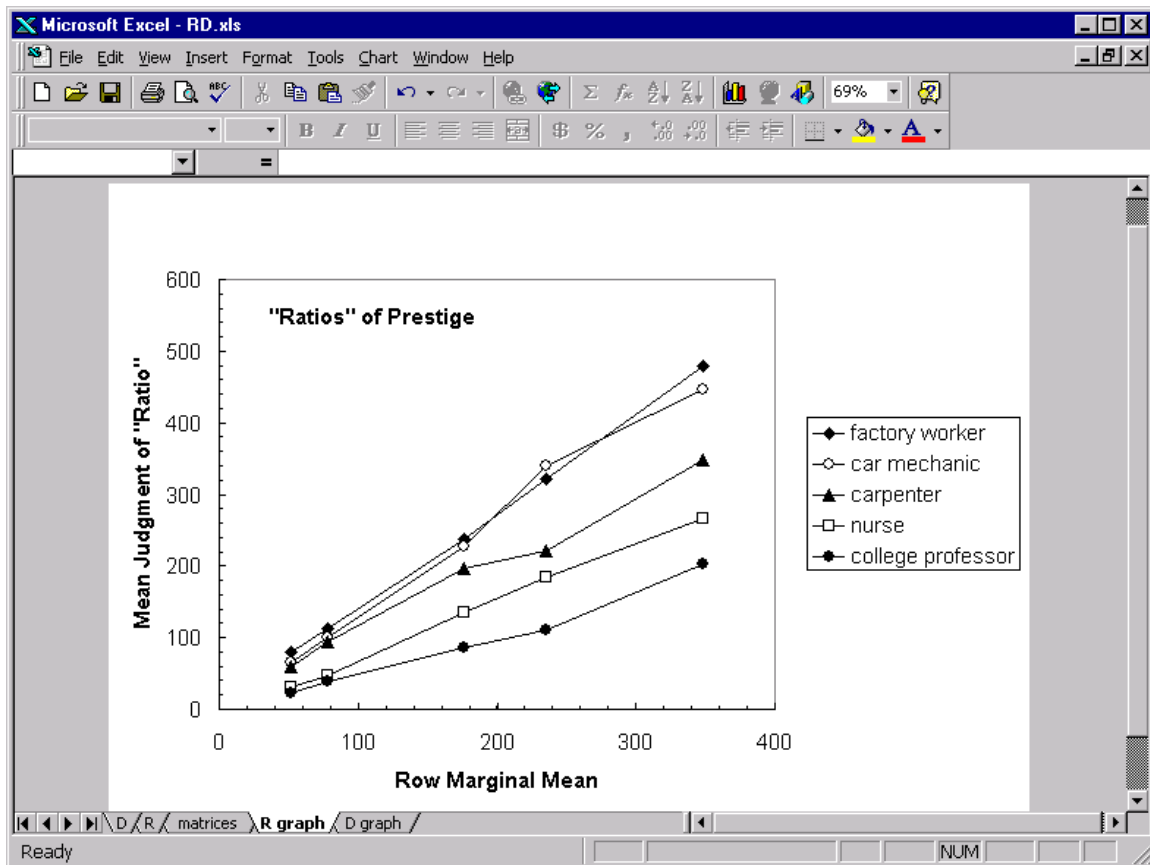
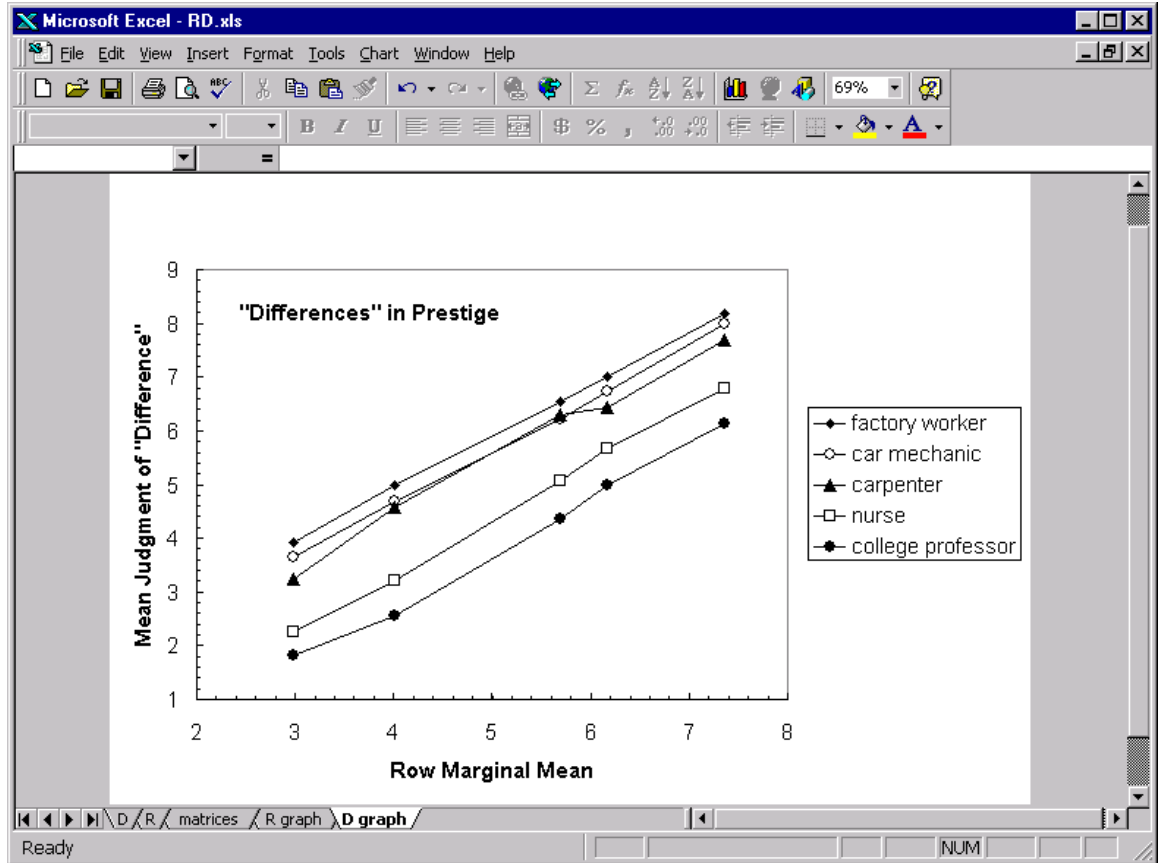


Figure 15.6. Mean judgments of “differences,” plotted as in Figure 15.5.



## H. Fit the Two-Operation Theory

The two-operation theory assumes that people compute both ratios and differences on the same scales of subjective value. To fit the theory, we need to estimate the subjective values of the prestige of the occupations. We can use the *Solver* in Excel to estimate the parameters of the theory.

To fit the two-operation theory, click on the tab for *matrices*, then carry out the following steps: (1) Select Cells A10:A11 and click the merge and center icon on the toolbar (it looks like the letter “a” with arrows in a box). Type the title “Two Operations.” (2) In Cells A11, A12, and A13, type the parameter labels  $a_R=$ ,  $b_R=$ , and  $m=$ . (3) In Cells B11, B12, and B13, type the corresponding initial values of these parameters; that is, type 1, 0, and 1, respectively. (4) Click in Cell B11 and give it the name  $A\_R$  by replacing B11 in the Name Box (the Name Box is directly over Column A in the spreadsheet), and then hit enter. Give B12 the name  $B\_R$ ; next, give B13 the name  $M\_$ . (5) Next, type the label “scale values” in B14. (6) In B15 through B19 type the initial row scale values; that is, type the numbers 1, 2, 3, 4, and 5. Select these cells (B15:B19) and rename them  $s\_i$ . (7) Now, type the initial values for the column scale values in Cells C14:G14, and name them  $t\_j$ . Now, in Cell C15, type the following:

$$=A\_R*(s\_i/t\_j)^{M\_} + B\_R$$

Then hit return. The value 1 will appear in C15. Next, click in C15 and use *AutoFill* to fill the first column, then while the first column is selected drag to the right to fill the entire matrix of predicted ratios. The matrix will now appear as in Figure 15.7.

Insert Figure 15.7 about here.





The next step is to construct the matrix of theoretical differences. First, use *Copy* and *Paste Special (Paste Link)* to copy the row scale values from B15:B19 to K15:K19. Second, use *Copy* and *Paste Special (Paste Link)* to copy the column scale values from C14:G14 to L14:P14. Name the row scale values in K15:K19 s\_iD; name the column scale values in L14:P14 t\_jD. In J11 and J12 type the labels aD and bD. In K11 and K12 type the initial estimates; that is type 1 and 5, respectively. Assign the name a\_D to K11 and assign the name b\_D to K12. In L15, type the following formula for the subtractive model:

$$=a\_D*(s\_iD - t\_jD) + b\_D$$

Hit return, and 5 will appear in L15. Select L15 and use *AutoFill* to fill in the first column, then the entire five by five matrix.

Next, compute the sum of squared differences between the judgments of “differences” and the predicted differences by typing =SUMXMY2(L3:P7,L15:P19) in Cell Q21. (You can also do this by clicking in Cell Q21 and select *function* from the **Insert** menu. Then select the SUMXMY2 function and specify the matrix of data and the matrix of predictions). To the left of Q21, select Cells M21:P21 and click the *Merge and Center* icon (looks like a little “a” in a box). Then type in that large merged cell “Sum of squared deviations =”. The predictions will appear as in Figure 15.8.

Insert Figure 15.8 about here.



In H21, compute the sum of squared deviations between the judged “ratios” and theoretical ratios by the same method. This time, however, divide the sum of squared residuals by 10000 (i.e., in Cell H21, type  $\text{=SUMXMY2}(C15:G19,C3:G7)/10000$ ). The reason to divide by the square of 100 is to put the “ratio” deviations on a scale comparable to that for “differences” (recall that “ratio” judgments are 100 times the subjective “ratio”). Next, in H22, add the sum of H21 and Q21 (type  $\text{=H21+Q21}$  in Cell H22 and hit enter). Label this as the “total sum of squared residuals”. The number in Cell H22 measures the badness of fit of the model to both arrays of data. The larger the number the worse the fit. We can use the *Solver* to make this number smaller. See Figure 15.9.

Insert Figure 5.9.



The next step is to use the *Solver* to estimate the parameters. Click in H22. Then from the **Tools** menu, select *Solver...* Be sure that Cell H22 is selected as the Target. Click the radio button to *Minimize*. Select the scale values and parameters of the model. These values are located in Cells B15:B19, Cells C14:G14, K11:K12, and B11:B13. To select these, click the *By Changing Cells:* box; then, while holding the Ctrl button, use the mouse to select the cells that contain parameters of the model. These cells hold the row and column scale values, and the constants of Equations 15.1 and 15.3. Next, click the *Add* button on the Solver Parameters dialog. This button is used to add constraints. Constrain the first row scale value to 1 by selecting  $B15 = 1$ . Similarly, we will initially take “ratios” at face value by constraining the ratio model constants to be 100, 0, and 1, respectively. The instructions also specified that if there was “no difference” that the “difference” judgment was to be 5, so constrain K12 to be 5. This dialog box is shown in Figure 15.10. Insert Figure 15.10 about here.

Click *Solve* on the button, and the values in the theoretical matrix will change. Excel has varied the parameters of the model to try to make the theory fit the data as well as possible. Clicking ok to keep the solver solution will show the changes in Figure 15.11. The total sum of squared residuals dropped from 32.22 (for the initial estimates) to 4.83 for the two-operation theory. This fit is better, but we can do much better still.

Insert Figure 15.11 about here.

Figure 15.10. The Solver Parameters dialog box. This setup is used to fit the two-operation theory with “ratios” at face value and assuming that “5” on the “difference” scale corresponds to no difference. Later fit the theory that allows  $m$  to be free.

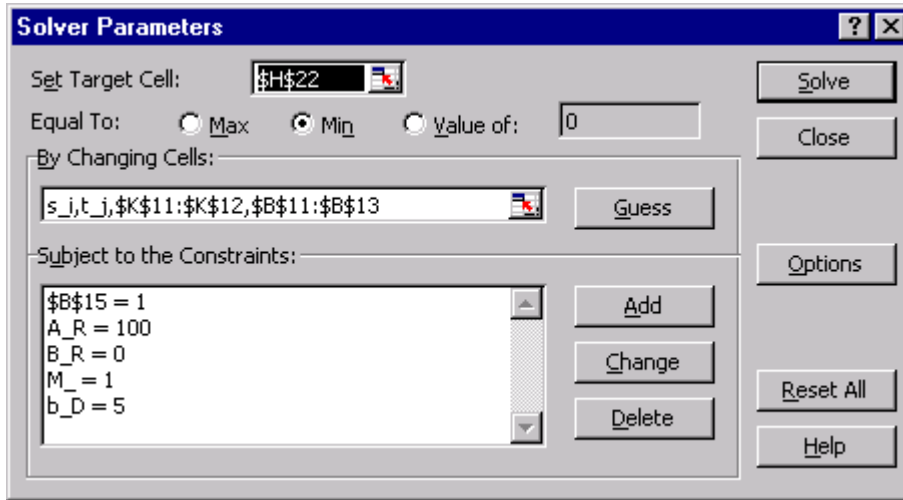


Figure 15.11. The results of the fit to two-operation theory with  $m = 1$ .

Microsoft Excel - RD.xls

File Edit View Insert Format Tools Data Window Help

Arial 10 B I U

H22 =H21+Q21

	A	B	C	D	E	F	G	H	I
8	col means		246.097	235.783	183.909	132.450	92.385		
9									
10	two operations								
11	aR=	100							
12	bR=	0							
13	m=	1							
14	scale values		5.002242	5.178432	7.184401	12.31691	16.2892		
15		1	19.991	19.311	13.919	8.119	6.139		
16		4.55039585	90.967	87.872	63.337	36.944	27.935		
17		12.8753126	257.391	248.633	179.212	104.534	79.042		
18		15.7361112	314.581	303.878	219.032	127.760	96.605		
19		22.9320575	458.436	442.838	319.192	186.184	140.781		
20									
21								Sum of squared deviations=	2.861807
22								Total sum of squared residuals=	4.830293
23									
24									
25									
26									
27									
28									

matrices R graph D graph

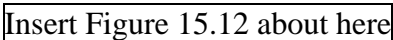
Ready NUM



## I. Fit of One-Operation Theory

To fit one-operation theory, you must create two new theoretical matrices. Put them below those for two-operation theory. Follow the same steps as before to construct the ratio model predictions, but this time give the row and column scale values the names  $s1_i$  (rows) and  $t1_j$  (columns) instead of  $s_i$  and  $t_j$ . The new names are used to distinguish these one-operation scale values from the two-operation scale values estimated previously. The equation in C29 is now:

$$=A1\_R*2^{(s1\_i-t1\_j)}+B1\_R$$

where the values of  $A1\_R$  and  $B1\_R$  are stored in Cells B25 and B26, respectively. Note that this equation is a subtractive model with an exponential transformation; in this case, the base is 2. This work is shown in Figure 15.12. 

Construct the predictions for the “difference” judgments according to the subtractive model, as was done previously, but give new names to the row and column scale values and parameters, to allow Excel to distinguish between the one and two-operation models. Then compute the sum of squared residuals between the theory and the data and add them together as before, putting the total in Cell H36.

Click in Cell H36 and select *Solver* from the **Tools** menu. Click Reset All, and specify the new row and column scale values and parameters as before. Also, constrain the parameters and one of the scale values to 1, as before. The *Solver* Parameter dialog is shown in Figure 15.13. Click the *Solve* button, click *Continue* if necessary, then click OK to keep the solution.

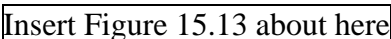


Figure 15.12. Subtractive model of “ratio” judgments for the one-operation theory.

Microsoft Excel - RD.xls

File Edit View Insert Format Tools Data Window Help

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C29 =A1\_R\*2^(s1\_i-t1\_j)+B1\_R

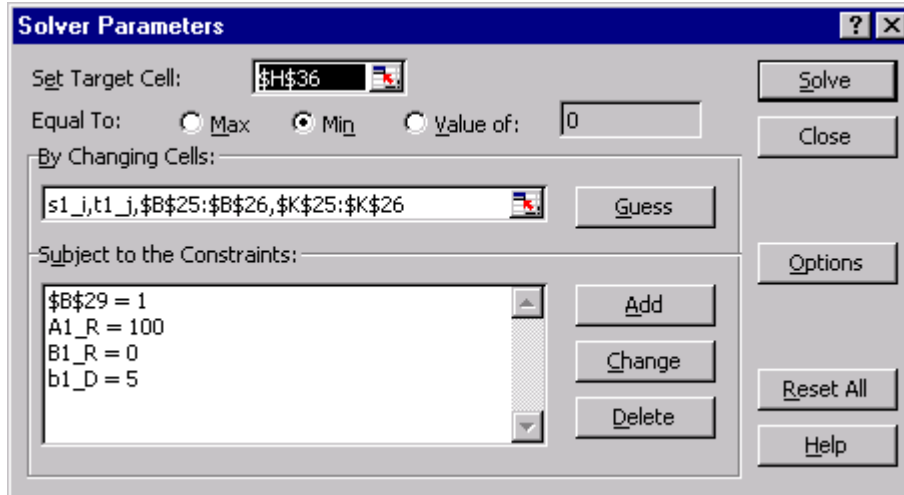
	A	B	C	D	E	F	G	H	I
18		1.20376929	312.392	290.666	234.476	149.837	104.941		
19		1.26096665	490.758	456.627	368.354	235.389	164.859		
20									
21								Sum of squared deviations=	1.063845
22								Total sum of squared residuals=	1.748677
23									
24	one operation theory								
25	aR=	100							
26	bR=	0							
27									
28	scale values		1	2	3	4	5		
29		1	100	50	25	12.5	6.25		
30		2	200	100	50	25	12.5		
31		3	400	200	100	50	25		
32		4	800	400	200	100	50		
33		5	1600	800	400	200	100		
34									
35								Sum of squared deviations=	170.458
36								Total sum of squared residuals=	184.9646
37									
38									

Subtractive model of "ratio" judgments

matrices R graph D graph

Ready NUM

Figure 15.13. Solver dialog box for one-operation theory.



The one-operation theory fits better than two-operation theory if ratios are taken at face value. The total sum of squared residuals is 1.74, less than half as large as 4.83.

The solution is shown in Figure 15.14. Insert Figure 15.14 about here.

We can improve the fit of the two-operation theory if we allow  $m$  to be free. Indeed, as  $m$  gets large, the two-operation theory can be made to approximate the predictions of one-operation theory (Birnbaum, 1980). To fit the two-operation theory with  $m$  free, repeat the steps above, except delete the constraint that  $m = 1$ . This model, which has one extra parameter, fits with a total sum of squared residuals of 1.75, about the same as the fit of one-operation theory. The best-fit value of  $m = 9.73$ . This solution implies that it is only 26% more prestigious to be a *physician* than a *trash collector*, and the ratio of prestige of a *physician* to a *factory worker* is 1.178, but people report this “ratio” to be about “five” because of the exponent,  $m = 9.73$  ( $1.178^{9.73} = 4.92$ ).

Because the extreme value of  $m$  seems inelegant and because the best-fit solution of two-operation theory is basically an approximation of one-operation theory, the one-operation theory seems a better representation of these data. It is simpler to assume that people use one operation than to assume that they use two, but due to an extreme value of an extra parameter, the data appear to be produced by one operation. Certainly the one-operation theory fits better than the theory of two operations with the assumption that ratios can be taken as face value. This conclusion is similar to that reached in previous research, where it has been found that for heaviness, loudness, pitch, and a variety of other continua, that one-operation theory describes judgments of “ratios” and “differences” in subjective value (Birnbaum, 1980; 1982; Hardin & Birnbaum, 1990; Mellers, Davis, & Birnbaum, 1984).

## **J. Summary**

Judgments of “ratios” and “differences” can be used separately to scale subjective magnitude. By studying both tasks in the same study, it is possible to test theories of how people compare stimuli. Data of many studies suggest that both judgments are mediated by subtraction, indicating that subjective value can be measured on an interval scale.

This chapter explained how to do a type of meta-analysis, in which two or more studies are combined. The usual application of meta-analysis in psychology is to combine general experiments to estimate the size of an effect. In contrast, in this chapter, the purpose of combining two studies was to test theories that can only be tested with two or more experiments. By combining data from “ratio” and “difference” tasks, you found that one-operation theory gave a better account of the data than two-operation theory. You also derived a scale of the prestige of occupations that reproduces judgments in two tasks.

Figure 15.14. Solution to one-operation theory. This theory fits better than two-operation theory with  $m$  fixed to 1. According to these data, one-operation theory provides the following scale values for occupations: *trash collector* (1.0), *plumber* (1.78), *factory worker* (1.86), *car mechanic* (1.96), *carpenter* (2.26), *nurse* (2.89), *college professor* (3.41), *architect* (3.49), and *physician* (4.15). Both “ratios” and “differences” are based on differences in these values.

The screenshot shows a Microsoft Excel spreadsheet titled "RD.xls". The spreadsheet contains a table of scale values for various occupations and the results of a subtractive model of "ratio" judgments. The table is organized into columns A through I. The data is as follows:

	A	B	C	D	E	F	G	H	I
25	aR=	100							
26	bR=	0							
27									
28	scale values		1.856	1.960	2.260	2.892	3.407		
29		1.000	55.257	51.398	41.758	26.951	18.856		
30		1.775	94.586	87.976	71.479	46.133	32.277		
31		3.068	231.735	215.542	175.122	113.027	79.080		
32		3.491	310.718	289.006	234.810	151.550	106.033		
33		4.154	491.803	457.437	371.656	239.873	167.828		
34									
35								Sum of squared deviations=	0.996018
36								Total sum of squared residuals=	1.736557
37									
38									
39									
40									
41									
42									
43									
44									
45									

The spreadsheet also shows the formula bar with the formula  $=H35+Q35$  and the status bar at the bottom indicating "Ready" and "NUM".

**K. Exercises**

1. Use Excel to draw a graph of the predicted “differences” according to one-operation theory. Do the predictions resemble the data in Figure 15.6?
2. Use Excel to draw a graph of predicted “ratios” according to one-operation theory. Do the predictions fit the observed judgments in Figure 15.5?
3. Draw a scatterplot, connected by straight lines, that shows the row marginal means of “ratios,”  $\bar{R}_{i\cdot}$  plotted against the row marginal means of “differences,”  $\bar{D}_{i\cdot}$ . This graph will have 5 points. Draw this graph for:
  - a. The mean judgments.
  - b. Predictions of one-operation theory.
  - c. Predictions of two-operation theory with “ratios” taken at face value.
  - d. Predictions of two-operation theory with parameters free.
4. Draw a scatterplot that shows  $R_{ij}$  on the ordinate, plotted against  $D_{ij}$  on the abscissa. For this experiment, each scatterplot will have 25 points. Plot this graph for:
  - a. Mean judgments (of “ratios” and “differences” of occupational prestige).
  - b. Predictions of one-operation theory.
  - c. Predictions of two-operation theory with “ratios” taken at face value; i.e., constrain  $m = 1$ ,  $a = 100$ , and  $b = 0$ .
  - d. Predictions of two-operation theory with  $m$ ,  $a$ , and  $b$  free.

5. Project idea: Devise a study in which people will show evidence of two operations for “ratios” and “differences.” Consider using intervals between stimuli (“distances”) as the stimuli.

6. Project idea. Ask Ss to judge “ratios” and “differences” of intervals of prestige. Analyze the data for evidence of one or two operations.