

## Issues in Utility Measurement

MICHAEL H. BIRNBAUM

*Irvine Research Unit in Mathematical Behavioral Science  
California State University, Fullerton*

Descriptive theories of decision making are constrained by the need to explain the following behavioral phenomena: risk aversion, gambling, purchases of insurance, investment, the paradoxes of Allais and Ellsberg, intransitivity of preference, irregularities of choice, preference reversals, risk judgments, violations of branch independence, the difference between buying and selling prices, violations of monotonicity, and the relationships between risky and riskless situations. The papers in this special issue on utility theory address these empirical phenomena and explore theories proposed to explain them. © 1992 Academic Press, Inc.

The study of human decision making has become a study of how people depart from theories that stipulate how one ought to make decisions. A rational decision maker should choose the course of action that is anticipated to lead to the best results, but there is some room for disagreement about the definitions of "anticipated" and "best." In recent years, psychologists have concluded that people consistently violate principles of decision making that have been held to be "rational." This special issue presents a collection of papers that test implications of psychological theories proposed to explain the following behavioral phenomena.

### "RISK AVERSION"

Most people prefer the expected value of a gamble to the gamble itself. Indeed, most would prefer \$40 for certain, rather than take a 50-50 gamble to receive \$100 or \$1, even though the gamble has a higher expected value (\$50.50). The fact that people have definite preferences among gambles of constant expected value led to the development of the theory of utility. If utility is a logarithmic function of money, as proposed by Bernoulli, then the utility of \$10 should match the expected utility of the 50-50 gamble between \$1 and \$100. A negatively accelerated utility function would also explain why people will pay more than the expected loss to buy insurance.

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Expected utility theory was formalized by von Neumann and Morgenstern (1947) and extended to events that did not have specified, objective probabilities by Savage (1954), who developed subjective expected utility theory. Evaluations of these theories and extensions are presented in Edwards (1954), Fishburn (1983), Keeney and Raiffa (1976), Krantz, Luce, Suppes, and Tversky (1971), Payne (1973), Raiffa (1968), Schoemaker (1982), Slovic, Lichtenstein, and Fischhoff (1988), and von Winterfeldt and Edwards (1986).

Equation (1) specifies a class of nonconfigural utility theories:

$$U = \sum s(p_i)u(x_i), \quad (1)$$

where  $U$  represents the overall utility of a gamble to receive outcome  $x_i$  with probability  $p_i$ ;  $s(p_i)$  and  $u(x_i)$  are subjective weights of probability and utility of outcomes; and the summation is over all mutually exclusive and exhaustive outcomes ( $\sum p_i = 1$ ). When  $s(p) = p$ , Eq. (1) reduces to expected utility theory; when  $u(x) = x$  and  $s(p) = p$ , it reduces to expected value theory. Equation (1) is called "nonconfigural" because the functions,  $s(p)$  and  $u(x)$ , are independent of the other outcomes and probabilities in each gamble. Despite the ability of Eq. (1) to explain risk aversion and the flexibility offered by two unspecified functions, experimental evidence is accumulating against the theory that one can predict actual decisions, bids, or ratings from Eq. (1).

### RISKY VS RISKLESS UTILITY

Expected utility theory leads to scales of the utility of money that disagree with estimates of utility based on riskless judgments (Bell & Raiffa, 1988; Hershey, Kunreuther, & Schoemaker, 1982; von Winterfeldt & Edwards, 1986; Tversky, 1967; Stevens, 1975; Stevenson, 1986). For example, the same subject might judge a 50–50 gamble to receive either \$0 or \$96 to be worth \$24, yet judge the "difference" in utility between \$96 and \$24 to exceed the "difference" in utility between \$24 and \$0.

Some investigators assumed that expected utility theory is true and postulated that there are two scales, "utility" and "value," for risky and riskless situations, respectively (e.g., Bell & Raiffa, 1988). Others proposed that utility should be an invariant construct and argued against expected utility theory. For example, Birnbaum and Sutton (1992) found that a rank-dependent, configural-weight model yields a scale of money that is compatible with buying and selling prices and also with scales estimated from judgments of "ratios" and "differences" in the riskless utility of monetary amounts. Because the scales derived from Eq. (1) were different for buying and selling prices and also differed from scales that fit riskless judgments, Birnbaum and Sutton argued against noncon-

figural theories in favor of a theory that can explain all of the data with a single utility function.

Shanteau and Troutman (1992) address a problem in riskless utility that has long interested economists and psychologists, the utility function for numbers of a good. Many economic texts begin with an imagined case of two people, for example, shipwrecked sailors on an island. One of them has one commodity, say cans of drink, and the other has another, say bags of food. Each person is willing to trade some of what he has to obtain some of the other's goods. The conclusion that both people want to exchange is a consequence of negatively accelerated scales of utility: The first unit of the new good is worth more than the last unit of the plentiful good. Thurstone (1931) tested a particular theory of "riskless utility." Riskless utility, however, became controversial among economists once it was realized that interesting results could be derived without assuming specific theories of utility indifference (see e.g., Ferguson, 1966, pp. 11–25).

Shanteau and Troutman (1992) review some of this controversy and test the theory that the shape of diminishing marginal return is the same for all goods. Their test makes use of Shanteau's (1974) method for assessing the multiplicative model. Because the multiplicative model defines a meaningful scale, if the model is successful it becomes an empirical question whether riskless utility, so defined, is the same construct as utility derived from risky choices.

Stevenson (1992) explores whether different scales are needed for risky and riskless investments that play out over time. Most people would prefer a positive outcome immediately rather than a larger one at a later time; only when interest rates are high enough will people invest. One aspect of long-term investment (besides having one's money tied up) is that risks of loss accumulate over time (Stevenson, 1986). Stevenson (1992) investigates how the discounting effects of time depend on risk and on context. She develops a set of equations to show how time and probability trade off.

### GAMBLING AMONG THE "RISK AVERSE" AND ALLAIS' PARADOX

Multiplication of the probabilities by a constant can reverse the rank order of preference between gambles (Allais, 1979; Kahneman & Tversky, 1979; Schoemaker, 1982), contrary to expected utility theory. For example, many people prefer \$3000 for sure to a gamble with a .8 chance to win \$4000; however, (multiplying both chances by .25), the same people prefer a .2 chance at \$4000 over a .25 chance at \$3000.

People who ordinarily seem "risk averse" sometimes appear "risk seeking." For example, people who would not pay expected value to play

a 50–50 gamble will pay more than expected value to buy a state lottery ticket that offers a tiny probability of a large payoff.

Such paradoxical changes from what was interpreted as “risk aversion” to “risk seeking” could be explained by a version of subjective expected utility theory, in which the subjective weights of probabilities don’t match objective probabilities and need not sum to one (Edwards, 1954; Kahneman & Tversky, 1979). Such results could also be explained by the theory that the utility function itself depends on the lottery (Becker & Sarin, 1987).

In lottery-dependent utility theory, the basic premise of an expectation is preserved at the expense of permitting the utility function to change in different lotteries. Allowing the utility function to depend on the lottery seems a natural extension of the theory that “utility” changes to “value” when probabilities become zero or one. By restricting the ways in which utility can change across situations, one can test the theory. Daniels and Keller (1992) compare the predictive accuracy of expected utility theory and lottery-dependent expected utility theory in a choice-based procedure.

Currim and Sarin (1992) also compare the accuracy of lottery-dependent against expected utility and weighted utility theory as well, using several techniques. Currim and Sarin (1992) review other evidence relevant to this assessment and conclude that when the theories are fit to data and the parameters are used to predict to new data, the simpler, expected utility theory is as accurate as the more complex theories, if not more so. As noted by Currim and Sarin, with a different specification and perhaps improved assessment techniques, lottery-dependent theory might perform better.

### ELLSBERG’S PARADOX

Choices between ambiguous events do not conform to subjective expected utility theory (Ellsberg, 1961). For example, suppose there are 100 balls in an urn, which contains 33 red balls and 67 balls that are either blue or green. Suppose one ball will be drawn at random, and payoffs will be made only as follows: Gamble R pays \$100 if a red ball is drawn; gamble G pays \$100 if a green ball is drawn; gamble RB pays \$100 if either a red or blue ball is drawn; gamble GB pays \$100 if either a green or blue ball is drawn. Many people prefer gamble R to G and GB to RB, contrary to subjective expected utility theory, which requires that the choice between red and green should be independent of the payoff for blue.

### BUYING AND SELLING PRICES

Buying and selling prices assigned to gambles and goods are farther apart than predicted by utility theory (Knetch & Sinden, 1984; Harless,

1989). They also differ in rank order; people will offer to pay more for N than W, but demand more to sell W than N, where W is a binary gamble with a wide range of outcomes and N is a gamble with slightly smaller expected value and a smaller range of outcomes (Birnbbaum & Stegner, 1979; Birnbbaum & Sutton, 1992; Birnbbaum, Coffey, Mellers, & Weiss, 1992). These changes in rank order can be explained by the theory that the configural weight of the lower-valued outcome changes as a function of the judge's point of view (buyer vs seller), but the utility function is unchanged in different viewpoints.

Configural-weight theory was developed to explain violations of independence in judgment experiments (Birnbbaum, 1974, 1982; Birnbbaum & Stegner, 1979). Rather than attributing attitude toward risk entirely to the utility function, risk aversion or risk seeking can be produced in part by configural weighting (Birnbbaum *et al.*, 1992). Configural-weight theory is closely related to rank-dependent utility theories (Chew, Karni, & Safra, 1987; Lopes, 1990; Luce, 1986, 1991; Luce & Narens, 1985; Miyamoto, 1989; Quiggin, 1982; Wakker, 1989, in press; Yaari, 1987; see also Machina, 1982), which were developed independently. Rank-dependent utility theories can accommodate violations of independence because the weight of an outcome can depend on its rank among the other outcomes as well as its probability.

### VIOLETIONS OF BRANCH INDEPENDENCE

Branch independence is the assumption that if two gambles have a branch in common (the same outcome at the same probability), then the effect of other outcomes should be independent of the value of that common branch. Birnbbaum *et al.* (1992) found that a .5 probability to win \$80 or \$8 was judged higher on the average than a .5 probability to win \$80, a .4 chance to receive \$16, and a .1 chance to get \$1. However, when the common branch was changed from \$80 to \$0, the order of judgments reversed. Branch independence is implied by nonconfigural theories, such as Eq. (1), but not by configural-weight theories (Birnbbaum & Stegner, 1979; Birnbbaum *et al.*, 1992; Lynch, 1979), which allow the weight of an outcome to depend on the other outcomes in each gamble.

### JUDGMENTS OF RISK

Usually, people prefer gambles that seem lower in risk, but judgments of risk seem to involve something separate from attractiveness. The idea that risk might be a distinct psychological construct that might help us understand preferences for gambles has been explored by Coombs and Lehner (1984), Luce and Weber (1986), Nygren (1977), Weber (1984), and Keller, Sarin, and Weber (1986), among others.

Weber, Anderson, and Birnbbaum (1992) compare judgments of risk and

attractiveness in order to elucidate the relationships between these two kinds of judgments and also to test between configural and nonconfigural theories of risk and attractiveness. A key aspect of their paper is to test whether the effect of a given branch depends on the number, value, and variance of the other outcomes.

### VIOLATIONS OF MONOTONICITY

In judgment tasks, where gambles are judged separately on different trials, prices can violate dominance (monotonicity). People assign a higher average price to the gamble with a .95 probability to win \$96, otherwise \$0, than they do to the gamble with the same chance to win \$96, otherwise \$24, even though the latter gamble dominates the former (Birnbbaum *et al.*, 1992; Mellers, Weiss, & Birnbbaum, 1992). When offered a direct choice, however, they rarely choose the dominated gamble (Birnbbaum & Sutton, 1992). Because monotonicity is so compelling, one might be tempted to conclude that if judgments violate this principle and preferences satisfy it, then choice should be the preferred method for assessing human behavior. However, choices can violate transitivity, whereas numerical judgments always satisfy this property.

### INTRANSITIVITY OF PREFERENCES

Preferences are not always transitive. It is possible to select gambles so that some subjects will tend, predictably, to choose A over B, B over C, and C over A (Tversky, 1969). Systematic violations of transitivity would rule out large classes of theories, but they are apparently considered exceptions to the rule. When confronted with evidence of their intransitivity, people seem to consider it an error, as they do violations of monotonicity in judgment. Subsequent theories investigated by Tversky have assumed transitivity (Kahneman & Tversky, 1979; Tversky & Kahneman, 1986; Tversky, Sattath, & Slovic, 1988).

### IRREGULARITY OF CHOICE

Choice proportions show a contextual effect in which the probability of selecting A from the set [A, B, C] can exceed the probability of choosing A from the set [A, B]. Huber, Payne, and Puto (1982) used alternatives, A and B, which were nearly indifferent, and then introduced a new alternative, C, which was dominated on all dimensions by alternative A, but which was intermediate between A and B on other dimensions.

### METHODS OF ELICITATION AND PREFERENCE REVERSALS

Gambles appear to have different values when assessed by different methods (Hershey *et al.*, 1982; Hershey & Schoemaker, 1985; von Win-

terfeldt & Edwards, 1986). For example, a subject asked to establish a certainty equivalence for a 50–50 gamble to receive \$200 or \$0 may say that it is worth \$50. However, when asked later to assign a probability equivalence,  $p$ , such that \$50 for sure is indifferent to the gamble to win \$200 with probability  $p$ , otherwise \$0, a consistent subject should say “.5,” but typically says, “.4.”

Schoemaker and Hershey (1992) distinguish three possible causes for such inconsistencies. The random noise theory attributes the effect to random error since the typical result can be described as “regression” from the first to the second task. The anchoring interpretation assumes that certainty equivalents are averages of “true” values and anchors presented in the experiment. The reframing hypothesis assumes that the judge recodes the outcomes in the probability equivalence task into deviations from the certainty value, which becomes an “aspiration level,” or zero point. Schoemaker and Hershey fit their results as a combination of these factors.

The preference order of gambles also changes in different tasks. A “preference reversal” is said to occur when subjects assign a higher price or judgment to gamble A than P, but prefer gamble P to A in a direct comparison.

In the “classic” preference reversal (Lichtenstein & Slovic, 1971; Lindman, 1971), the gambles have equal expected value, and gamble A has a smaller probability to win a larger amount, whereas gamble P has a higher probability to win a smaller amount. Ratings of the attractiveness of gambles and the prices assigned to them also reverse rank order. People will often rate P higher than A, but will assign a higher price to A than to P. Preference reversals have received considerable attention in recent investigations (e.g., Bostic, Herrnstein, & Luce, 1990; Goldstein & Einhorn, 1987; Grether & Plott, 1979; Karni & Safra, 1987; Slovic & Lichtenstein, 1983; Tversky *et al.*, 1988).

Goldstein and Busemeyer (1992) explore a lexicographic criterion model as an explanation of changes in rank order produced by experimental conditions. This model generalizes the lexicographic semiorder used by Tversky to account for intransitive preferences. Different experimental conditions are theorized to affect internal response processes (as opposed to evaluation of the stimuli). Techniques for testing the model are illustrated in an application to data.

Busemeyer and Goldstein (1992) also address the problem of preference reversals, from the viewpoint of decision field theory. In that theory, preferences are the result of an aggregation of considerations favoring one alternative or the other in a dynamic, stochastic process. The theory allows predictions of decision times as well as choice proportions, and the

theory is applied to pricing judgments as well. A key concept is the variance of subjective comparisons, an idea that is also useful for understanding violations of scalability (Busemeyer, 1985).

Mellers, Ordóñez, & Birnbaum (1992) test three theories of preference reversals between ratings and prices. Contingent weighting theory (Tversky *et al.*, 1988) assumes that the process of combining probability and outcomes is invariant, but the weights of these factors change in different tasks. Expression theory (Goldstein & Einhorn, 1987) assumes that an implicit judgment process is affected by the within-gamble context. Change of process theory (Mellers *et al.*, 1992) assumes that the scales of probability and utility are invariant, but different tasks and contexts induce the judge to utilize different processes for combining the scales.

### CONTEXTUAL EFFECTS

In judgment research, it is well known that the judgment of a given stimulus depends on the other stimuli presented; however, contextual effects have received less attention in decision making (Parducci, 1968). Mellers *et al.* (1992) investigate whether utility functions and/or the judgment processes depend on the distribution of other gambles offered for judgment. Mellers *et al.* (1992) find that when most of the other gambles are lower in expected value, a gamble will be judged higher than when it is presented with gambles of higher expected value (see also Mellers & Birnbaum, 1983). Stevenson (1992) manipulates the range of time durations to investigate whether the effect of a given delay depends on the other delays presented in the study.

Mellers *et al.* (1992) also report that the rank order of judgments can be altered by the choice of other gambles presented. In one context, gamble A is judged higher than gamble P; however, when new gambles (with zero and near-zero values of probability and amount) are also presented for judgment, gamble P is judged higher than gamble A. This contextual experiment was suggested by the change of process theory.

### UNIFYING THEMES AND THEORETICAL DIFFERENCES

The papers in this issue all deal with problems that arise in applying nonconfigural theories [e.g., Eq. (1)] to human judgments, but the authors approach these exceptions from different theoretical perspectives. One important difference concerns the nature of utility. Stevenson (1992), Currim and Sarin (1992), and Daniels and Keller (1992) reason that utility may change as the risk of the situation changes; however, Weber *et al.* (1992) and Mellers *et al.* (1992) retain the concept of an invariant utility function. The papers also differ in the invariances (besides utility) they try to preserve: linearity in probability, weights, aggregation process, or response process. Because the papers in this issue treat different phenom-



ena, these alternate theoretical stances do not always stand in direct contradiction.

Like the blind philosophers who felt different parts of an elephant and argued over its true nature ["It is like a wall." (side) "It is like a snake." (trunk) "It is like a tree." (leg) "It is like a rope." (tail)], the authors in this issue present different perspectives on utility. The reader is invited to study these views to decide whether utility is like that deity the ancient Celts were loathe to name (who changes shape whenever observed), or whether it is really like a single, invariant elephant that is merely too large to grasp in one examination.

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