Research Article

VIOLATIONS OF BRANCH INDEPENDENCE IN JUDGMENTS OF THE VALUE OF GAMBLES

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Abstract—*Branch independence is weaker than Savage's "sure thing" principle. It requires that judgments of gambles with a common outcome produced by the same probability-event must not reverse order when that common outcome is changed. Subjects judged 168 gambles from viewpoints of both buyer (highest buying price) and seller (lowest selling price). Judgments violated branch independence in both viewpoints. Violations also changed systematically between viewpoints, consistent with the theory that viewpoint affects configural weighting but not the utility function. Violations of branch independence were opposite those predicted by the model of cumulative prospect theory. The middle of three equally likely outcomes received the most weight in the seller's viewpoint. In the buyer's, lower outcomes received greater weights. In both viewpoints, the ratio of weights of the middle outcome to the highest outcome exceeded the ratio of weights of the lowest outcome to the middle outcome.*

Expected utility (EU) theory (von Neumann & Morgenstern, 1947) and subjective expected utility (SEU) theory seemed to offer not only a prescriptive theory (Savage, 1954) of how a rational decision maker should operate, but also a descriptive theory of how people actually do make decisions. SEU theory contains two psychological scales: subjective probability, to represent beliefs concerning the contingency of outcomes on actions, and utility, to describe subjective values that people place on outcomes.

Although normative theorists treated outcomes as final states of wealth, descriptive theorists (e.g., Edwards, 1954, 1962; Kahneman & Tversky, 1979; Markowitz, 1952) considered the *utility* (or *value*) function to be a psychological scale of changes from a reference point. Psychologists also treated the scale of probability as a weighting function that need not obey the algebra of probability.

Psychological SEU theory represents the utility of a choice of action as the weighted average of the utility of the consequences (Edwards, 1954). The SEU of a gamble can be written as follows:

$$\mathrm{SEU} = \sum s(p_i)u(x_i), \tag{1}$$

where $s(p_i)$ is the subjective (weight of) probability of outcome x_i, p_i is the objective probability, and $u(x_i)$ is the utility (or psychological value) of receiving x_i. Because this formulation uses two free functions, $s(p)$ and $u(x)$, Equation 1 is flexible enough to resist refutation in most experiments. It has not yet been replaced by a rival for applied decision making (Edwards, 1992).

Savage's (1954) SEU theory rests on and implies Savage's Axiom, called the "sure thing" principle. This principle asserts that if two options yield the same outcome for a state of nature, then the value of that common consequence should not affect the decision.

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The Allais paradox (Allais, 1979) and Ellsberg paradox (Ellsberg, 1961) are sometimes cited as evidence against Savage's Axiom (e.g., Slovic & Tversky, 1974). However, several authors have disputed whether these paradoxes and their variations are direct tests of Savage's Axiom (Cohen & Jaffray, 1988; Luce, 1992; Stevenson, Busemeyer, & Naylor, 1991). Concerning the Allais paradox, Edwards (1954) remarked, "One way of avoiding these difficulties is to stop thinking of a scale of subjective probabilities and, instead, to think of a weighting function applied to the scale of objective probabilities" (p. 398).

Equation 1, which allows such weighting, implies branch independence, a weaker form of Savage's Axiom. Branch independence requires that if two gambles have a common branch (the same outcome produced by the same event with the same known probability), then the preference order induced by other components of the gambles will be independent of the value of the common outcome. The present research investigated branch independence to test SEU formulations against configural models and to distinguish among configural theories.

In the past 25 years, investigators from different perspectives have converged on the thesis that outcomes do not combine their effects independently, but rather that the weight of an outcome depends on its relationships to other outcomes in the same set. Following previous usage (Birnbaum, 1973b, 1974), we use the term *configural weighting* to refer to models in which the weight of an item depends on its rank order among the other components. Luce and Narens (1985) showed that a purely rank-dependent representation is the most general two-outcome model that implies interval scales of utility. Models in which the weights can depend on the ranks accommodate many otherwise puzzling phenomena in decision making (Birnbaum & Sotoodeh, 1991; Birnbaum & Stegner, 1979; Birnbaum & Sutton, 1992; Birnbaum, Thompson, & Bean, in press; Lopes, 1990; Luce, 1990, 1992; Luce & Fishburn, 1991; Machina, 1982; Miyamoto, 1989; Quiggin, 1982; Tversky & Kahneman, 1992; Wakker, 1993, 1994; Wakker, Erev, & Weber, 1994; Weber, 1994). These models have in common that weights can depend on the ranks, but they make different predictions for tests of stochastic dominance and branch independence (Birnbaum & McIntosh, 1996; Birnbaum, in press).

BRANCH INDEPENDENCE IN THREE-OUTCOME GAMBLES

The present experiment used lotteries composed of three equally likely outcomes, denoted (x, y, z). A can contained three slips that were identical except that different numbers were written on them. The slips were mixed, and one was drawn blindly at random to determine the outcome. If Slip 1 was chosen, the outcome was x; if Slip 2 was chosen, the outcome was y; if Slip 3 was chosen, the outcome was z.

The SEU for such a three-outcome gamble can be written:

$$\mathrm{SEU}(x, y, z) = s(p)u(x) + s(p)u(y) + s(p)u(z), \tag{2}$$

where SEU(x, y, z) represents the SEU of the gamble; $s(p)$ is the weight of $p = 1/3$; and $u(x)$, $u(y)$, and $u(z)$ represent utilities of the outcomes.

In this case, branch independence can be written:

$$\begin{gathered}(x, y, z) \text{ is judged higher than } (x', y', z)\\ \text{if and only if}\\ (x, y, z') \text{ is judged higher than } (x', y', z').\end{gathered} \tag{3}$$

In other words, replacing the common outcome z with z' should not affect the rank order of judgments between (x, y) and (x', y'). In this case (fixed probabilities), branch independence is equivalent to joint independence (Krantz, Luce, Suppes, & Tversky, 1971).

Equation 2 implies branch independence. Suppose judgments are a strictly monotonic function of SEU in Equation 2; then (x, y, z) will be judged higher than (x', y', z) if and only if

$$s(p)u(x) + s(p)u(y) + s(p)u(z) > s(p)u(x') + s(p)u(y') + s(p)u(z);$$

subtracting $s(p)u(z)$ from both sides and adding $s(p)u(z')$ to both sides, we have

$$s(p)u(x) + s(p)u(y) + s(p)u(z') > s(p)u(x') + s(p)u(y') + s(p)u(z'),$$

which implies (x, y, z') should be judged higher than (x', y', z'); that is, the order is independent of the common branch.

RANK-DEPENDENT CONFIGURAL WEIGHT THEORIES

For this experiment, rank-dependent utility (RDU) can be written as follows:

$$\mathrm{RDU}(x, y, z) = w_L u(x) + w_M u(y) + w_H u(z) \tag{4}$$

for $0 < x < y < z$, where w_L, w_M, and w_H are the configural weights of the lowest, medium, and highest of three equally likely outcomes, respectively.

Equation 4 implies violations of branch independence (contradicting Expression 3), where $0 < z < x' < x < y < y' < z'$, whenever either

$$\frac{w_L}{w_M} < \frac{u(y') - u(y)}{u(x) - u(x')} < \frac{w_M}{w_H}, \tag{5a}$$

in which case RDU(z, x, y) > RDU(z, x', y') but RDU(x, y, z') < RDU(x', y', z'), or

$$\frac{w_L}{w_M} > \frac{u(y') - u(y)}{u(x) - u(x')} > \frac{w_M}{w_H}, \tag{5b}$$

in which case RDU(z, x, y) < RDU(z, x', y') but RDU(x, y, z') > RDU(x', y', z'). In other words, branch independence will be violated whenever the ratios of weights "straddle" the ratio of differences in utility (Birnbaum & McIntosh, 1996). If the weights are equal (as they are in SEU theory), or if they stand in any fixed ratio (e.g., 4/7: 2/7: 1/7), there will be no violations of branch independence in this situation.

If the response is a judged price, and if the utility function is approximated as a power function, $u(x) = x^b$, then Equation 4 can be revised as follows:

$$P = a[w_L x_L^b + w_M x_M^b + w_H x_H^b]^{1/b} + c \tag{6}$$

where P is the predicted judgment; a and c are linear constants; b is the exponent of the utility function; 1/b is the exponent of the inverse function from RDU to money; x_L, x_M, and x_H are the lowest, middle, and highest outcomes, respectively; and w_L, w_M, and w_H are the corresponding relative weights, constrained so that $w_L + w_M + w_H = 1$.

To illustrate how Equation 6 violates branch independence, let us use it to evaluate the following pairs of gambles:

($4, $39, $45) vs. ($4, $12, $96)
($39, $45, $136) vs. ($12, $96, $136).

The gambles in the first pair share a common branch with z = $4, whereas the second pair have z' = $136. Suppose w_L, w_M, and w_H are .50, .35, and .15, respectively, conforming to Expression 5a (.5/.35 < .35/.15). Let a = b = 1 and c = 0. Predicted judgments for the first two gambles are then

$$P(\$4, \$39, \$45) = \$22.4 > P(\$4, \$12, \$96) = \$20.6; \tag{7a}$$

however, the second two have the opposite order,

$$P(\$39, \$45, \$136) = \$55.6 < P(\$12, \$96, \$136) = \$60.0, \tag{7b}$$

which is a violation of branch independence (Expression 3).

Cumulative prospect theory (CPT), as modeled by Tversky and Kahneman (1992), assumes that the middle outcome of three equally likely positive outcomes receives the least weight. If the middle outcome has the least weight, Expression 5b follows; therefore, CPT predicts the opposite pattern of violations for any decumulative weighting function that assigns the least weight to the middle outcome. According to the parameters of Tversky and Kahneman, the weights are .487, .177, and .336, respectively (.487/.177 > .177/.336; Expression 5b). With a = 1, c = 0, and $u(x) = x^{.88}$, CPT yields preferences opposite those of Expressions 7:

$$P(\$4, \$39, \$45) = \$22.8 < P(\$4, \$12, \$96) = \$33.3; \tag{8a}$$

and

$$P(\$39, \$45, \$136) = \$71.1 > P(\$12, \$96, \$136) = \$65.2. \tag{8b}$$

These predictions violate branch independence and illustrate that different configural weights can produce opposite patterns of violations.

The judge's *point of view* refers to payoffs or instructions that bias the relative cost to the judge of over- versus underestimation (Birnbaum & Stegner, 1979). Previous research is consistent with the conclusion that configural weighting depends on the judge's point of view. In the buyer's viewpoint, more weight is placed on lower valued outcomes or estimates of value; in the seller's viewpoint, relatively more weight is placed on higher and middle values than lower values (Birnbaum, Coffey, Mellers, & Weiss, 1992; Birnbaum &

McIntosh, 1996; Birnbaum & Stegner, 1979; Birnbaum & Sutton, 1992; Birnbaum & Zimmermann, 1995).

In sum, if SEU holds, and the utility function changes in different viewpoints, then the order of judgments of the gambles can change between points of view, but there will be no violations of branch independence. If rank-dependent configural weight theory holds, however, there can be violations of branch independence. If point of view affects configural weighting, then viewpoint should change violations of branch independence predictably. The exact pattern of violations can distinguish different configural weight utility models.

METHOD

Instructions

In the buyer's viewpoint, subjects judged "the most a buyer should pay to buy the chance to play the lottery." They were told that the buyer exchanges money for the opportunity to play the lottery.

In the seller's viewpoint, subjects judged the "least that a seller should accept to sell the lottery . . . the seller receives money and gives up the chance to play the lottery." Additional instructions are in Birnbaum and Sutton (1992).

Stimuli

Each lottery was displayed as in the following example:

1. _____($4, $12, $96)

This display represents a gamble with equal chances (p = 1/3) of winning $4, $12, or $96.

Design

There were 168 gambles consisting of three equally likely outcomes (x, y, z), constructed from a 6 × 28 factorial design with six levels of z combined with 28 (x, y) pairs. The levels of z were $2, $4, $35, $49, $124, and $148. The 28 levels of ($x$, y) pairs, listed within each range ($|x - y|$) in descending order of total value ($x + y$), were as follows:

- range = $0: ($54, $54), ($36, $36), ($24, $24)
- range = $6: ($51, $57), ($45, $51), ($39, $45), ($33, $39), ($27, $33), ($21, $27), ($15, $21)
- range = $12: ($48, $60), ($42, $54), ($36, $48), ($30, $42), ($24, $36), ($18, $30), ($12, $24)
- range = $24: ($42, $66), ($12, $36)
- range = $36: ($36, $72), ($12, $48)
- range = $48: ($30, $78), ($12, $60)
- range = $60: ($24, $84), ($12, $72)
- range = $72: ($18, $90), ($12, $84)
- range = $84: ($12, $96)

Totals ($x + y$) ranged from $36 to $108 in steps of $12, with at least two levels of range for each total.

Procedure

The 168 combinations were printed in random order in booklets, with the restriction that the same outcome not appear on consecutive trials. Buyer's and seller's booklets were identical (describing both points of view), except the final paragraph in each booklet explained which task (buyer's or seller's) the subject was to perform first and was followed by eight warm-up trials in that viewpoint. The second task contained another summary of that task and another set of warm-ups in the new viewpoint. Half of the groups received the buyer's task first, and half received the seller's task first. Subjects completed both tasks within 2 hr.

Subjects

Subjects were 46 undergraduates who were enrolled in introductory psychology.

RESULTS

Figure 1 illustrates a pattern of violation of branch independence that was repeated with other combinations. Judgments of buyer's price for the gamble (z, $12, $96) are plotted against outcome z as open circles (dashed curve shows corresponding predictions from Equation 6, discussed in the next section). The wide-range gamble crosses the

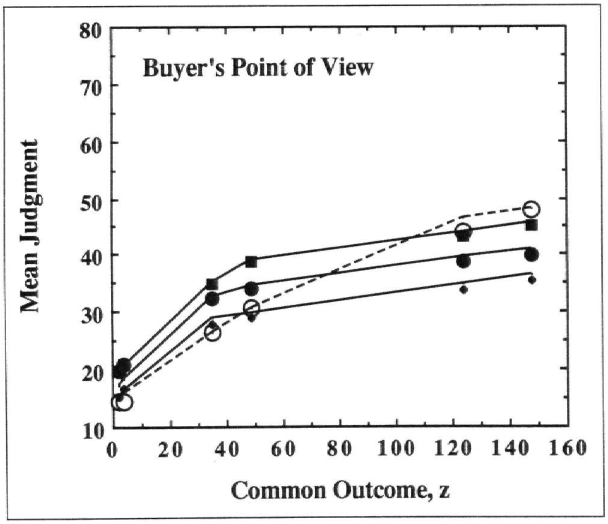

Fig. 1. Mean judgments of gambles from the buyer's viewpoint, plotted as a function of the common value, z. Crossovers of curves represent violations of branch independence. Open circles show judgments of (z, $12, $96); the dashed curve shows predictions for (z, $12, $96) based on rank-dependent configural weight theory, with $u(x) = x$. Filled diamonds, circles, and squares show mean judgments for (z, $27, $33), ($z$, $33, $39), and ($z$, $39, $45), respectively (solid lines show corresponding predictions). Note that the open circles are below all solid symbols for $z \leqslant \$35$ but are above all solid symbols for $z \geqslant \$124$, violating branch independence.

narrow-range gambles, violating branch independence. For example, the judgment of (z, \$12, \$96) is less than the judgment of (z, \$39, \$45) when $z \leq \$49$, but the order reverses when $z \geq \$124$.

Figure 2 shows that a similar pattern of violations occurs in the seller's viewpoint. Judgments of (z, \$12, \$96) have a steeper slope as a function of z than judgments of (z, \$39, \$45). However, crossovers occur for different narrow-range gambles in Figures 1 and 2. In Figure 2, (z, \$45, \$51) and (z, \$51, \$57) cross as well, even though they strictly exceed (z, \$12, \$96) in the buyer's viewpoint. Similarly, (z, \$27, \$33) crosses in Figure 1, but is strictly below (z, \$12, \$96) in the seller's viewpoint. The pattern in Figure 2 was repeated for other combinations in the seller's viewpoint, and it was characteristic of the majority of individual subjects. The pattern in Figures 1 and 2 agrees with Expressions 5a and 7, but is opposite that predicted by the cumulative prospect model (Expressions 5b and 8).

Figure 3 shows mean buying prices for gambles in which $x + y = 108$, plotted against $|x - y|$. For each level of z, judgments decrease as $|x - y|$ increases.

Figure 4 shows mean selling prices for the same gambles as in Figure 3. Mean judgments again decrease as a function of $|x - y|$ when $z < y$; however, when $z > y > x$, judgments increase with increasing range. The reversal of order (for different values of z) violates branch independence. Note that for $z \leq \$49$, ($z$, \$51, \$57) is judged higher than (z, \$12, \$96); however, for $z \geq \$124$, the order is reversed. Out of 46 subjects, 35 showed this pattern of violations of branch independence for the seller's viewpoint; 7 of the remaining 11 subjects had all negative slopes, resembling the buyer's viewpoint.

The changing slopes in Figures 3 and 4 can be explained by configural weight theory with $u(x) = x$, if in the buyer's viewpoint, $w_L > w_M > w_H$, and in the seller's viewpoint, $w_L < w_M > w_H$. When z is the highest outcome (upper curves in Fig. 4, $z \geq \$124$), the curves will increase if $w_M > w_L$. When z is lowest, x and y are the middle and highest; hence, the curves will decrease if $w_M > w_H$. Figures 3 and 4 show that changing the viewpoint changes the pattern of violations of

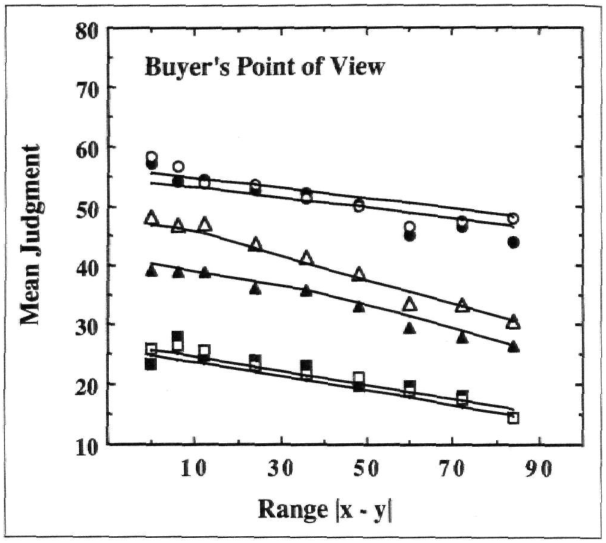

Fig. 3. Mean judgments of buyer's price for gambles in which $x + y = 108$, plotted as a function of $|x - y|$. Filled squares, open squares, filled triangles, open triangles, filled circles, and open circles show results for z = \$2, \$4, \$35, \$49, \$124, and \$148, respectively. Lines show corresponding predictions for rank-dependent configural weight theory, with $u(x) = x$.

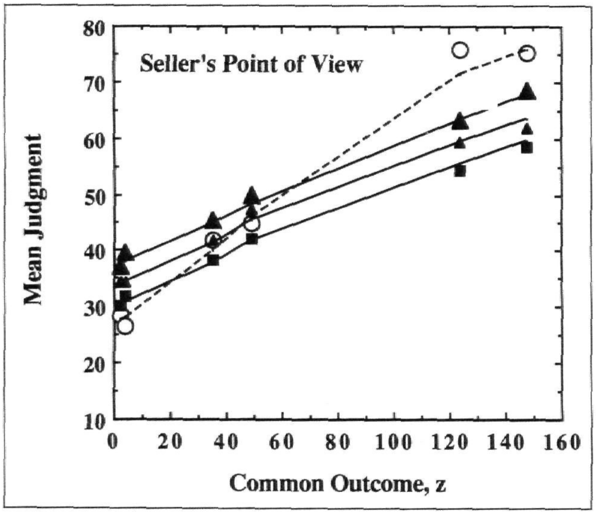

Fig. 2. Violations of branch independence in the seller's point of view, plotted as in Figure 1. Open circles show judgments of (z, \$12, \$96); the dashed curve shows corresponding predictions based on rank-dependent configural weight theory, with $u(x) = x$. Solid squares and small and large triangles show judgments of (z, \$39, \$45), (z, \$45, \$51), and (z, \$51, \$57), respectively; solid lines show corresponding predictions.

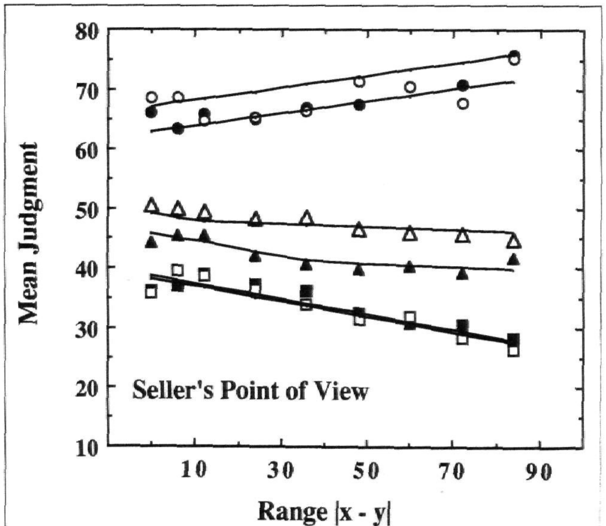

Fig. 4. Mean judgments of seller's price for gambles in which $x + y = 108$, plotted as a function of $|x - y|$. Filled squares, open squares filled triangles, open triangles, filled circles, and open circles show results for z = \$2, \$4, \$35, \$49, \$124, and \$148, respectively. Lines show corresponding predictions for rank-dependent configural weigh theory, with $u(x) = x$.

Table 1. *Estimated parameters of Equation 6 and index of fit*

b	c	a	w_L	w_M	w_H	Sum of squared residuals
			Buyer's viewpoint			
1.31	3.48	0.87	.64	.31	.05	247.2[a]
(1)	3.94	0.84	.56	.36	.08	273.2
(1)	(0)	0.93	.52	.39	.09	656.8
			Seller's viewpoint			
1.37	3.79	0.89	.34	.50	.16	519.4[a]
(1)	4.45	0.84	.27	.52	.21	557.1
(1)	(0)	0.92	.25	.53	.22	1046.9

Note. Values in parentheses are fixed; w_L, w_M, and w_H are the configural weights of the lowest, medium, and highest outcomes, respectively. Standard errors of the weights are less than .02 in all cases when b = 1.
[a]The sum of squared residuals for the general model with b and c free corresponds to 1% and 1.3% of the total variance for buyer's and seller's judgments, respectively.

branch independence in a manner that can be explained by a change in configural weights.

Fit of RDU Theory

Equation 6 was fit separately in each point of view. Least-squares parameter estimates are given in Table 1 for models in which both b and c were free, c was free but b was fixed, and both b and c were fixed.

The version of Equation 6 with b = 1 and c free fits nearly as well as the general version with both b and c free, consistent with Birnbaum et al. (1992); this version correlated .995 and .993 with the buyer's and seller's mean judgments, respectively. This version was used to generate the predictions shown as curves in Figures 1 through 4, and this model (with b = 1) was also used to conduct the analyses presented in Tables 2 and 3.

The models were fit to individual subjects with similar results. With b = 1, median weights were .50, .28, and .04 in the buyer's viewpoint and .20, .41, and .16 for the seller's, for aw_L, aw_M, and aw_H, respectively. In the buyer's viewpoint, w_L is significantly greater than w_M, which is significantly greater than w_H. However, in the seller's viewpoint, w_M is significantly greater than the other two, which are not significantly different from each other.

In both viewpoints, estimated weights satisfy Expression 5a: $w_L/w_M < w_M/w_H$. This relation predicts the pattern of violations in Figures 1 and 2. These weights also explain the changing slopes in Figure 4. Predictions in Figures 1 through 4 appear to provide a good fit, using the same $u(x)$ function in both viewpoints. One exception is evident in Figures 3 and 4: The data show a wider gap due to z than predicted when $|x - y| = 0$, suggesting that the weight of two equal outcomes is less than the sum of their weights when unequal.

Predicting Violations of Branch Independence

Because correlations can be "high" despite systematic deviations (Birnbaum, 1973a), we performed a more precise evaluation of the model's ability to predict violations of branch independence. First, rank orders of the mean judgments were determined for the 28 (x, y) pairs within each value of z. Then, 15 matrices of differences in rank order were calculated between each of the 15 combinations of z and z'. According to SEU theory, the rank order in each matrix should be the same, so there should be no differences except for random error (violations of branch independence should be small and unpredictable). According to rank-dependent configural weight theory, changes in rank order of data should be predictable from changes in rank order of predictions. In the special cases in which changes in z keep the rank order of the outcomes the same (comonotonic), Equations 4 and 6 require no ordinal change (Wakker et al., 1994).

Table 2 shows the variances of the differences for the data (upper

Table 2. *Variances of violations of branch independence (changes in rank order due to changes in* z*)*

Value of z	Value of z': 2	4	35	49	124	148
2		2.89 *0.00*	5.00 *0.44*	4.96 *1.63*	10.44 *11.26*	11.80 *11.26*
4	2.00 *0.00*		2.37 *0.44*	4.22 *1.63*	12.00 *11.26*	12.94 *11.26*
35	4.67 *4.37*	6.74 *4.37*		2.37 *1.93*	14.07 *13.56*	14.24 *13.56*
49	4.28 *9.41*	6.28 *9.41*	4.02 *2.81*		10.07 *8.22*	10.57 *8.22*
124	25.26 *33.11*	32.74 *33.11*	19.70 *20.30*	19.24 *13.04*		0.54 *0.00*
148	29.80 *33.11*	33.80 *33.11*	21.65 *20.30*	19.67 *13.04*	6.94 *0.00*	

Note. The upper entry in each cell is the observed variance of changes in rank order; the lower entry is the predicted variance (italicized). Data above the diagonal are for the buyer's point of view; data below the diagonal are for the seller's.

Table 3. *Correlations between predicted and obtained changes in rank order (violations of branch independence) due to variation of common outcome from z to z'*

	Value of z'					
Value of z	2	4	35	49	124	148
2		.00	.53	.27	.80	.77
4	.00		.29	.10	.88	.85
35	.57	.57		.31	.90	.90
49	.61	.70	.29		.87	.84
124	.92	.89	.90	.88		.00
148	.83	.86	.79	.81	.00	

Note. Buyer's results are shown above the diagonal; seller's results are shown below the diagonal. The critical value of r is .463 for $\alpha = .01$.

value) and the predictions (lower value), respectively. The model predicts no change in rank order between z = \$2 and z = \$4, and it also predicts no change between z = \$124 and z = \$148. If the data were perfectly consistent with the model, the rank orders should be identical, so the variances in these comonotonic cases would be zero. The comonotonic changes in z produce relatively small variances of the obtained changes in rank order (median = 2.4). In contrast, the largest observed (and predicted) variances of differences in rank order are between z = \$2, \$4, or \$35 and z = \$124 or \$148 (noncomonotonic changes). Variances changing from z lowest to z' highest have medians of 11.9 and 31.3 in the buyer's and seller's viewpoints, compared with predictions of 11.3 and 33.1, respectively. Correlations between the variances of predicted and obtained changes in rank order are .97 and .94 for buyer's and seller's data, respectively.

Table 3 shows correlations between predicted changes in rank order and obtained changes in rank order as z is changed to z'. There are 420 potential violations of branch independence, composed of 15 pairs of z and z' by 28 (x, y) pairs. The pooled correlations, between obtained changes in rank order and predicted changes in rank order (the ability of the model to predict these violations), are .97 and .94 for buyer's and seller's judgments, respectively. For all pairs of z and z' for which the common outcome changes from lowest to highest, correlations range from .77 to .92, with a median of .86.

The correlation between differences in rank order of the mean judgments of the 168 gambles between buyer's and seller's viewpoints and differences in rank order of the predictions is .94.

In sum, SEU theories fail to explain the present data because they cannot account for violations of branch independence. The weighting function of CPT also fails to account for the present data because its predictions correlate negatively with the observed changes in rank order. The rank-dependent configural weight model (Equation 6) predicts both the variances and the directions of violations of branch independence with reasonable accuracy, as well as changes in rank order due to point of view.

DISCUSSION

Violations of Branch Independence in Judgment

The results provide strong evidence of violations of branch independence. Figures 1 through 4 are not consistent with SEU, even allowing different utility functions for different points of view. Because branch independence is a weak form of Savage's sure-thing principle, violations of this property rule out Savage's SEU theory and Edwards's (1954) psychological SEU theory as descriptive of these data.

Instead, violations are consistent with the pattern predicted by rank-dependent configural weight theory. They can be predicted from different weights in each point of view, using the same $u(x)$ function in both viewpoints. The results are compatible with previous conclusions (Birnbaum & Stegner, 1979; Birnbaum & Sutton, 1992; Birnbaum et al., 1992) that configural weighting of lower outcomes is greater in the buyer's than in the seller's viewpoint. Equation 6 predicts violations of branch independence in Figures 1 and 2 and predicts changes in those patterns due to point of view (Figs. 3 and 4; Tables 2 and 3).

The data are consistent with the premise of scale convergence, the assumption that $u(x)$ is independent of viewpoint, configuration, and task (Birnbaum, 1974; Birnbaum & Sutton, 1992).

Violations of Branch Independence in Choice

Judgment and choice experiments do not always agree. Reversals of preference between these procedures can be quite complex (Birnbaum & Sutton, 1992; Mellers, Chang, Birnbaum, & Ordóñez, 1992; Mellers, Ordóñez, & Birnbaum, 1992). In this case, however, the pattern of violations in Figures 1 and 2 has also been observed in choice (Birnbaum & McIntosh, 1996). For example, 65% and 75% of subjects in two studies preferred (\$2, \$40, \$44) over (\$2, \$10, \$98); however, 61% and 60% of the same subjects preferred (\$10, \$98, \$108) over (\$40, \$44, \$108). These results for choice also imply that $w_L/w_M < w_M/w_H$. Because Expression 5a characterizes both buyer's and seller's viewpoints in judgment, it appears that judgment and choice have something in common, even though these procedures produce different preference orders.

Table 4 compares the weights estimated from the present judgment data against those estimated by Birnbaum and McIntosh (1996) for preferences between gambles. Weights for choice appear to be intermediate between weights for buyer's and seller's viewpoints in judgment, closer to the buyer's than the seller's. The fact that the same pattern of violations is observed in choice as in judgment suggests that

Table 4. *Estimated relative weights of three equally likely outcomes as a function of rank in three experiments*

	Rank of outcome		
Experiment	Lowest	Middle	Highest
Buyer's prices	.56	.36	.08
Seller's prices	.27	.52	.21
Preferences	.51	.33	.16

Note. Relative weights are normalized to sum to one by dividing by the sum of weights in each case. Values for preferences are based on the model of strength of preference judgments for nondominated choices in Birnbaum and McIntosh (1996). All three experiments are fit with the same utility function, $u(x) = x$, for $0 < x \leq \$148$.

the violations are not produced by something peculiar to only one method.

COMPARING RANK-DEPENDENT CONFIGURAL WEIGHT MODELS

The results in Figures 1 through 4 are not consistent with the CPT model of Tversky and Kahneman (1992), in which the middle outcome has the least weight. This assumption implies Expression 5b, which predicts a pattern of violation opposite that observed. The CPT model predicts that curves for $z < 12$ in Figures 3 and 4 should have positive slope because $w_M < w_H$ in CPT; the positive slopes in Figure 4 for $z \geqslant \$124$ are also contrary to the model. Figures 1 and 2 and the choice data of Birnbaum and McIntosh (1996) all show violations opposite the pattern predicted by the CPT model.

The model of CPT was fit to the inverse-S relation between certainty equivalents of binary gambles (to win x with probability p; otherwise, 0) and probability. The decumulative assumption of CPT requires that the middle of three equally likely outcomes receives the same weight as the increment produced by changing p from 1/3 to 2/3 in a binary gamble. Configural weight theory, in contrast, fits binary gambles with the relative weight assumption derived from asymmetric loss theory in Birnbaum et al. (1992, Equation 6). The relative weight of the higher outcome is given by

$$\frac{w_H f(p)}{w_H f(p) + w_L f(1 - p)}, \tag{9}$$

where w_L and w_H are configural weights (estimated by Birnbaum et al., in press, and Birnbaum et al., 1992, to be .63 and .37 in the neutral viewpoint), and $f(p)$ is a function of probability. According to the rank-dependent theory of Quiggin, the weight of $p = 1/2$ should be 1/2, so these studies are not consistent with Quiggin's theory (Birnbaum et al., in press). Expression 9, with $f(p) = p^{.56}$, and $u(x) = x$, makes predictions that are virtually identical to those of Tversky and Kahneman (1992) for binary gambles with $0 < x < \$150$.

CPT can probably be improved by giving up the theory that weights depend on cumulative or decumulative probability. An attraction of the cumulative representation is that it avoids violating stochastic dominance. However, it is an empirical question whether or not stochastic dominance is satisfied. Experiments on violations of monotonicity (Birnbaum, 1992, in press; Birnbaum & Thompson, 1996) indicate that stochastic dominance can be systematically violated with procedures similar to those of Tversky and Kahneman (1992) when certainty equivalents are determined by comparison of gambles against a fixed set of cash amounts. Because configural weight theory and CPT explain binary gambles equally well, but CPT fails to explain three-outcome gambles, configural weight theory seems preferable, given the present data. However, many implications of the theories remain to be tested.

Wakker et al. (1994) compared rates of comonotonic and noncomonotonic independence in a study of choice and did not find evidence requiring rejection of EU in favor of RDU. Weber and Kirsner (in press) modified the Wakker et al. experiment and observed small, but systematic violations favoring RDU, in a pattern that was consistent with the present results and those of Birnbaum and McIntosh (1996).

Wakker et al. (1994) noted that comonotonic independence is the key test between EU and RDU. That conclusion is certainly true for the standard RDU. However, configural weight theory derived from asymmetric loss functions (Birnbaum et al., 1992; Weber, 1994) can lead to different patterns of rank-dependent configural weighting, depending on the spacing of the outcomes as well as their ranks. Minimization of the squared loss function, for example, with different weights for over- and underestimation, leads to a model that can be equivalent to RDU in a restricted subdomain, but violates comonotonic independence over the entire domain (Birnbaum & McIntosh, 1996, Appendix A). Although the present results and those of Birnbaum and McIntosh (1996) are compatible with comonotonic independence, these studies have not really put comonotonic independence to a strenuous test. Investigation of comonotonic independence will distinguish alternative theories of configural weighting.

CONCLUSIONS

The present results, along with choice results of Birnbaum and McIntosh (1996), refute EU theory's account of risk aversion. According to utility theory, people prefer sure wins over gambles with positive expected values because the utility function for money is concave downward. RDU theory explains risk aversion as the consequence of heavy weights applied to low-valued outcomes. The present results show that risk aversion can be well fit using the approximation $u(x) = x$, if weights are allowed to depend on rank. Utility theories, flexible though they are, cannot account for violations of branch independence, but rank-dependent configural weight theory can predict such violations.

Although buying and selling prices are not monotonically related (Figs. 1 vs. 2, 3 vs. 4), these data can be well fit by the assumption that the utility function is invariant with respect to point of view, and only the weights are changed. The change in weights explains changes in the rank orders and changes in violations of branch independence.

Although the weights are quite different for different viewpoints, they have a property in common with each other and with choice: All three conform to Expression 5a, showing a pattern of violations of branch independence that is observed in all three tasks. This pattern is opposite that predicted by Expression 5b, which is implied by the weighting function of CPT.

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