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THE PARADOXES OF ALLAIS, STOCHASTIC DOMINANCE, AND DECISION WEIGHTS

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The purpose of this chapter is to compare the descriptive adequacy of alternative theories of decision making. The common consequence paradox of Allais, which is evidence against expected utility theory, can be interpreted as a joint test of branch independence (a weaker version of Savage's axiom), coalescing (equal outcomes can be combined by adding their probabilities), and transitivity. Thus, this paradox can be explained in several ways. One class of theories (including subjectively weighted utility theory and original prospect theory) retains branch independence but violates coalescing, and thereby violates stochastic dominance. Another class of theories (rank-dependent and rank- and sign-dependent utility theories including cumulative prospect theory) retains coalescing and stochastic dominance but violates branch independence. New independence properties, distribution independence and cumulative independence, are proposed to test original prospect theory and cumulative prospect theory. Violations of distribution independence refute original prospect theory and a multiplicative configural weight model. Experimental results also show violations of cumulative independence and stochastic dominance, contrary to rank-dependent utility theories, including cumulative prospect theory. Empirical results are consistent with a weight tax configural weight model that accounts for the Allais paradoxes, violations of branch and distribution independence, violations of cumulative independence, violations and satisfactions of stochastic dominance, and violations of coalescing.

PROBLEMS FOR EXPECTED UTILITY THEORY

By 1954, Expected Utility (EU) theory was in trouble as a descriptive theory of decision making. The EU model can be written as follows:

$$\text{EU}(G) = \sum p_i u(x_i) \quad (1)$$

where G is a gamble with probabilities p_i to win monetary outcomes x_i; EU(G) is the Expected Utility of gamble G; the summation is over all possible outcomes of the gamble; $\sum p_i = 1$; and $u(x_i)$ is the utility of outcome x_i. Let $\succ$ represent the preference relation between gambles. In a choice between two gambles, it is assumed that $G_1 \succ G_2$ if and only if $\text{EU}(G_1) > \text{EU}(G_2)$.

Edwards (1954, p. 393), in his review of EU theory wrote,

> "If this model is to be used to predict actual choices, what could go wrong with it? It might be that the probabilities by which the utilities are multiplied should not be the objective probabilities; in other words, a decider's estimate of the subjective importance of a probability may not be the same as the numerical value of that probability. It might be that the method of combination of probabilities and values should not be simple multiplication. It might be that the method of combination of the probability-value products should not be simple addition. It might be that the process of gambling has some positive or negative utility of its own. It might be that the whole approach is wrong…"

EU theory had several difficulties with data that were discussed by Edwards (1954) and later reviewed by Camerer (1989), Edwards (1992), Kahneman and Tversky (1979), Luce (1992), Schoemaker (1982), Starmer (1992), Stevenson, Busemeyer, and Naylor (1991), von Winterfeldt and Edwards (1986), and Wu and Gonzalez (1996). The most serious of these difficulties were the paradoxes of Allais (1953/1979), known as the *common ratio* and *common consequence* paradoxes. They were termed "paradoxes" because seemingly rational people were willing to defend choices that were in violation with EU theory.

Common Ratio Paradox

The common ratio problem can be illustrated by the following pair of choices:

Choice 1: Would you prefer A or B?

A:	$3,000 for sure	*B*:	.80 probability to win $4,000 .20 probability to win $0

Choice 2: Would you prefer A' or B'?

A':	.25 probability to win $3,000 .75 probability to win $0	*B*':	.20 probability to win $4,000 .80 probability to win $0

Most people prefer A over B in the first choice and B' over A' in the second choice, contrary to EU theory. This combination of preferences violates EU theory, which implies that people should choose either A and A' or B and B'.

According to EU theory, setting $u(0) = 0$, $A \succ B$ holds if and only if,

$$u(3000) > .8u(4000),$$

where $u(3000)$ and $u(4000)$ represent the utilities of \$3,000 and \$4,000, respectively. However, the second choice ($A' \prec B'$) implies,

$$.25u(3000) < .20u(4000).$$

Multiplying both sides of this inequality by 4 reveals a direct contradiction. Because the second choice is derived from the first by multiplication by a common factor (.25), these problems are known as common ratio problems.

EU theory implies *ratio independence*, which asserts that choices should be independent of the common ratio, a, as follows:

$$A = (x, p; 0, 1-p) \succ B = (y, q; 0, 1-q)$$

if and only if

$$A' = (x, ap; 0, 1-ap) \succ B' = (y, aq; 0, 1-aq).$$

where $(x, p; 0, 1-p)$ denotes a gamble that yields \$$x$ with probability p and \$0 otherwise. Because choices violate the property of ratio independence, they are considered paradoxical.

Common Consequence Paradox

Choices 3 and 4 illustrate the common consequence paradox:

Choice 3:

C:	\$.5 Million for sure	D:	.10 probability to win \$1 Million .89 probability to win \$.5 Million .01 probability to win \$0

Choice 4:

C':	.11 probability to win \$.5 Million .89 probability to win \$0	D':	.10 probability to win \$1 Million .90 probability to win \$0

Most people express the following preferences: $C \succ D$ and $D' \succ C'$ in problems of this type, contrary to EU theory. According to EU theory, a judge should prefer C and C' or D and D', but it is a contradiction to choose C and D' or D and C'. According to EU, where $u(.5\text{M})$ and $u(1\text{M})$ are the utilities of \$.5 Million and \$1 Million, $C \succ D$ if and only if,

$$u(.5\text{M}) > .10u(1\text{M}) + .89u(.5\text{M});$$

therefore, $$.11u(.5\text{M}) > .10u(1\text{M}),$$

which holds if and only if $C' \succ D'$, contrary to the empirical choice.

Common Consequence Independence

More generally, EU theory implies common consequence independence, which can be defined as follows:

$$C' = (x, p; 0, 1 - p) \succ D' = (y, q; 0, 1 - q)$$

if and only if

$$C = (x, p; z, r; 0, 1 - p - r) \succ D = (y, q; z, r; 0, 1 - q - r),$$

where (z, r) is the common branch. It is useful to define *Allais independence* as the special case of common consequence independence where $y > x = z > 0$, and the equal outcomes of x and z are coalesced. The Allais paradox occurs when a decision maker systematically violates Allais independence.

Analysis of the Allais Paradox

The common consequence paradox of Allais (1953) was presented as a test of Savage's "sure thing" or independence axiom (Allais, 1953/1979; Allais & Hagen, 1979; Slovic & Tversky, 1974). According to the "sure thing" principle, if two gambles give the same consequence for a given state of nature, then that consequence should have no effect on the choice between the two gambles. The common consequence paradox is evidence against EU; however, it can be explained by several different psychological theories. It is useful to analyze Allais independence into simpler components to understand possible psychological explanations for the effect.

Transitivity, Coalescing, and Branch Independence

Let $G = (x, p; y, q; z, r)$ represent a three-outcome gamble that yields a consequence of x with probability p, y with probability q, and z with probability $r = 1 - p - q$, where the probabilities are nonzero. Let $\succ$ represent the preference relation, and $\sim$ represent indifference.

1. *Transitivity of preference* means $A \succ B$ and $B \succ C$ implies $A \succ C$.

2. *Coalescing* means that equal outcomes can be combined by adding their probabilities; for example, for three outcome gambles,

$$(x, p; x, q; z, r) \sim (x, p + q; z, r) \text{ and } (x, p; y, q, y, r) \sim (x, p; y, q + r).$$

3. *Branch independence* requires that if two gambles have a common branch (the same outcome produced by the same event with known probability), then the preference between the gambles will be independent of the outcome on that common branch. The term "branch" also designates that the probability-outcome combination is distinct in the problem presentation.

For three outcome gambles, branch independence requires

$$(x, p; y, q; z, r) \succ (x', p'; y', q'; z, r)$$

if and only if

$$(x, p; y, q; z', r) \succ (x', p'; y', q'; z', r).$$

where (z, r) is the common branch, the outcomes (x, y, z, x', y', z') are all distinct, the probabilities are not zero and sum to 1 in each gamble. This principle is weaker than Savage's independence axiom because it holds for branches of known probability and also because it does not presume coalescing.

Common Consequence Independence Deduced

Common consequence independence can be derived as follows: $C \succ D$ is the same as ($.5M, 1) $\succ$ ($0, .01; $.5M, .89; $1M, .10); by coalescing, ($.5M, 1) ~ ($.5M, .01; $.5M, .89; $.5M, .10); by transitivity, ($.5M, .01; $.5M, .89; $.5M, .10) $\succ$ ($0, .01; $.5M, $.89; $1M, .10). By branch independence, the common branch ($.5M, .89) can be changed to ($0, .89); therefore, ($0, .89; $.5M, .01; $.5M, .10) $\succ$ ($0, .01; $0, .89; $1M, .10); coalescing equal outcomes on both sides, we have, ($0, .89; $.5M, .11) $\succ$ ($0, .9; $1M, .10), which is the same as $C' \succ D'$.

The Allais paradox thus contradicts the combination of transitivity, coalescing, and branch independence–the three properties used above to derive the conclusion that $C \succ D$ iff $C' \succ D'$. Therefore, it is possible to explain the paradox with a theory that satisfies branch independence but violates coalescing, as in Subjectively Weighted Utility (SWU) theory (Edwards, 1954; 1962; Karmarkar, 1978; 1979) and Original Prospect (OP) theory (Kahneman & Tversky, 1979). It is also possible to explain the paradox by a theory that retains coalescing but violates branch independence, as is done in rank-dependent utility (RDU) theories (Quiggin, 1982; 1985; Luce & Fishburn, 1991; 1995; Tversky & Kahneman, 1992). Finally, it may be that branch independence and coalescing are both violated, as is the case in configural weight utility (CWU) theories (Birnbaum & Stegner, 1979; Birnbaum & McIntosh, 1996; Birnbaum & Navarrete, 1997).

This analysis has an interesting relation to that of Savage (1954/ 1972, p. 101-103), who converted the Allais problem into a problem satisfying branch independence. He did this by devising a lottery in which there were 100 equally likely tickets, numbered from 1 to 100. The prize for ticket 1 was $.5M in lotteries *C* and *C*' versus $0 in lotteries *D* and *D*'; the prizes for tickets 2-11 were $.5M in lotteries *C* and *C*' versus $1M in lotteries *D* and *D*'; and the prizes for tickets 12-100 was either .5M in both *C* and *D*, or $0 in both *C*' and *D*'. Savage's analysis implicitly used coalescing and explicitly used a translation of probabilities into event-probability branches. Savage confessed that his own choices had been paradoxical until he conducted his analysis, and that after analysis his choices became consistent with the sure thing axiom.

Keller (1985) found that the incidence of paradoxical choices was less when the problems were presented in an uncoalesced format, similar to Savage's representation, than when the problems are presented in their usual, coalesced, verbal form. Perhaps the Allais paradox is due to a violation of coalescing, as predicted by subjectively weighted utility (SWU) models.

SWU MODELS ACCOUNT FOR ALLAIS PARADOXES

Edwards (1954) recognized that subjectively weighted utility models of the form,

$$\text{SWU}(G) = \sum S(p_i)u(x_i.) \qquad (2)$$

could account for observed choices, but that these models had problems of their own. A special case of this model in which $u(x) = x$ had been suggested by Preston and Baratta (1948). In early work, the $S(p)$ function was considered a psychophysical function that related subjective probability to objective probability. However, Edwards (1954; 1962) considered cases in which $S(p)$ was restricted to follow the laws of probability and also cases in which it was allowed to violate them.

According to SWU, the common ratio problem can be explained as follows: with $u(0) = 0$, the choice of A implies,

$$S(1)u(3000) > S(.8)u(4000)$$

and the choice of B' over A' implies,

$$S(.25)u(3000) < S(.20)u(4000).$$

It follows that,

$$S(1)/S(.8) > u(4000)/u(3000) > S(.25)/S(.2)$$

which is not a contradiction (though it would be if $S(p) = bp^{\gamma}$). In general, common ratio violations should occur, according to SWU, whenever the ratios of $S(p)/S(q)$ and $S(ap)/S(aq)$ "straddle" the ratio of utilities, $u(y)/u(x)$.

In the common consequence problem, according to SWU, $C \succ D$ if and only if

$$S(1)u(.5\text{M}) > S(.10)u(1\text{M}) + S(.89)u(.5\text{M})$$

Similarly, $D' \succ C'$ holds if and only if $S(.10)u(1\text{M}) > S(.11)u(.5\text{M})$.

Combining both preferences, it follows:

$$S(1) - S(.89) > S(.11); \text{ therefore, } S(1) > S(.89) + S(.11).$$

Edwards (1954) noted that if it were assumed that $S(1) = 1$, then results like these contradict the idea that the subjective probabilities of complementary events should sum to 1, suggesting that S(p) should not be regarded as a subjective probability. As Edwards (1954, p. 398) put it,

> "One way of avoiding these difficulties is to stop thinking about a scale of subjective probabilities and, instead, to think of a weighting function applied to the scale of objective probabilities which weights these objective probabilities according to their ability to control behavior… There is no reason why such weighted probabilities should add up to 1 or should obey any other simple combinatory principle."

Edwards' (1954) analysis of decision weights was extended in Edwards (1962) to allow weights to differ for different categories of prospects. Edwards (1962, p. 128) suggested a configural extension of the subjectively weighted utility model in which different weighting functions for probabilities of different events might require different pages in a book of weights,

> "The data now available suggest the speculation that there may be exactly five pages in that book, each page defined by a class of possible payoff arrangements. In Class 1, all possible outcomes have utilities greater than zero. In Class 2, the worst possible outcome (or outcomes, if there are several possible outcomes all with equal utility), has a utility of zero. In Class 3, at least one possible outcome has a positive utility and at least one possible outcome has a negative utility. In Class 4, the best possible outcome or outcomes has a utility of zero. And in Class 5, all possible outcomes have negative utilities."

Original Prospect (OP) theory (Kahneman & Tversky, 1979) is a special case of the model suggested by Edwards (1962), in which Classes 1 and 5 are collapsed into one category and Classes 2-4 into another category. As in Edwards' (1954, 1962) treatment, in OP theory, utility functions were defined as changes from a reference level, the framing or format of the problems is considered important, and Equation 2 was retained for up to two nonzero outcomes.

To understand how Equation 2 behaves, it helps to illustrate it with a numerical example. These examples will use the weighting formula of Lattimore, Baker, and Witte (1992),

$$S(p) = \qquad (3)$$

where c and γ are positive constants. This model assigns $S(0) = 0$ and $S(1) = 1$, and S is a strictly increasing monotonic function.

SWU (Equation 2), with Equation 3, assuming that $u(x) = x^{\beta}$, where β is the exponent of the power function, and with the values $c = \gamma = \beta = .4$ can account for the common ratio and common consequence paradoxes. For the common ratio problem, SWU(A) = 24.6 > SWU(B) = 11.33; furthermore, SWU(A') = 5.04 < SWU(B') = 5.16. For the common consequence problem, SWU(C) = 190.4 > SWU(D) = 127.16; additionally, SWU(C') = 28.12 < SWU(D') = 35.8. Thus, the model accounts for both the common ratio and common consequence paradoxes.

But that's not all that Equation 2 does; it also violates stochastic dominance (Fishburn, 1978).

Stochastic Dominance and SWU

For Gambles $G_1 \neq G_2$, G_1 *stochastically dominates* G_2 if and only if $P(x_i > t \mid G_1) \geq P(x_i > t \mid G_2)$ for all t, where $P(x_i > t \mid G_j)$ is the probability of receiving an outcome greater than t given Gamble G_j.

The statement, "choices satisfy stochastic dominance" means that if G_1 stochastically dominates G_2, then $G_1 \succ G_2$. It would be a violation of stochastic dominance when a judge prefers the dominated gamble. As shown below, stochastic dominance can be viewed as the combination of several simpler properties, including transitivity, coalescing, and outcome monotonicity (improving an outcome holding everything else constant should improve any gamble).

Equation 2 violates dominance in transparent situations such as Choice 5.

Choice 5:

E:	.5 probability to win \$100	*F*:	.99 probability to win \$100
	.5 probability to win \$200		.01 probability to win \$200

E clearly dominates *F* because the outcomes are the same, but the probability of the better outcome (\$200) is higher in *E* than in *F*. However, SWU(*E*) = 4.18 < SWU(*F*) = 5.01, so this model predicts a violation of dominance that few humans would commit. Similarly, consider Choice 6.

Choice 6:

G:	.5 probability to win \$110	*H*:	.01 probability to win \$101
	.5 probability to win \$120		.01 probability to win \$102
			.98 probability to win \$103

Clearly, *G* dominates *H* because all of its possible outcomes exceed all possible outcomes of *H*; however, SWU(*G*) = 3.81 < 4.94 = SWU(*H*), so this SWU model predicts that subjects should choose *H* over *G*! If SWU were to be retained as descriptive of empirical choices, it would have to be modified to avoid these predictions.

Editing Rules in Prospect Theory

In their paper on prospect theory, Kahneman and Tversky (1979) proposed a number of editing rules to avoid such unwanted predictions. In addition to defining utility with respect to changes from a status quo and adopting Edwards' (1954) concern for the psychophysics of the display or "framing" of the problem, prospect theory includes six additional editing principles to allow the subject to simplify gambles and choices between gambles prior to evaluation by the SWU equation. These editing principles are as follows:

1. Combination: probabilities associated with identical outcomes are combined. This principle corresponds to coalescing.

2. Segregation: a riskless component is segregated from the risky component.

> "the prospect (300, .8; 200, .2) is naturally decomposed into a sure gain of 200 and the risky prospect (100, .8) (Kahneman & Tversky, 1979, p. 274)."

3. Cancellation: Components shared by both alternatives are discarded from the choice.

> "For example, the choice between (200, .2; 100, .5; –50, .3) and (200, .2; 150, .5; –100, .3) can be reduced by cancellation to a choice between (100, .5; –50, .3) and (150, .5; –100, .3) (Kahneman & Tversky, 1979, p. 274-275)."

If subjects cancel common components, then they will satisfy branch independence and distribution independence, which will be taken up in a later section.

4. Dominance: Transparently dominated alternatives are recognized and eliminated. This principle eliminates the troublesome predictions for Choices 5 and 6 above.

5. Simplification: rounding off probabilities and outcomes.

6. Priority of Editing: Editing precedes and takes priority over evaluation. Kahneman and Tversky (1979, p. 275) remarked,

> "Because the editing operations facilitate the task of decision, it is assumed that they are performed whenever possible."

Without the editing operations, the algebraic model of prospect theory predicts dominance violations of the kinds that would not be descriptive of human behavior. Because *S*(p) is nonlinear, it is possible to take a certain branch and divide it into smaller pieces in such a way that the total weight can be increased, creating dominance violations. Another way to avoid some (but not all) violations of stochastic dominance is to use an averaging model instead of an additive model.

SUBJECTIVELY WEIGHTED AVERAGE UTILITY

The subjectively weighted average utility (SWAU) model can be written as follows:

$$\text{SWAU(G)} = \sum S(p_i)u(x_i)/\sum S(p_i) \qquad (4)$$

By dividing by the sum of the weights, the sum of relative weights $[S(p_i)/\sum S(p_i)]$ will be 1 within each gamble. This means that although $S(p)$ is a function of p, the relative weight of a given probability depends on the distribution of probabilities in the gamble. The models of Karmarkar (1979), Viscusi (1989), and Lattimore, et al. (1992) are of this type.

The SWAU model (Equation 4), with the $S(p)$ function of Equation 3, $u(x) = x^{\beta}$, and $c = \gamma = \beta = .4$, predicts the common consequence effect, the common ratio effect, and does not violate transparent stochastic dominance in Choices 5 and 6. For gambles A, B, A', and B', the predicted certainty equivalents (u^{-1}(SWAU(G)), are \$3,000 > \$1,566 and \$215.2 < \$218.8, respectively; for C, D, C', and D', they are \$500,000 > \$474,156, and \$13,432 < \$24,011; for E and F they are \$145 > \$106, and for G and H they are \$115 > \$103, respectively. Although Equation 4 satisfies dominance for Choices 5 and 6 (E versus F and G versus H), it does violate coalescing and stochastic dominance in other situations (Quiggin, 1985), which will be taken up in a later section.

Tversky and Kahneman (1986) discussed the issue of violations of dominance. They argued that stochastic dominance will be satisfied when it is "transparent," due to editing, but that it might be violated when the relation is "masked" by the framing of the problem. They reported a choice problem in which 58% of the subjects chose the dominated gamble. The dominance relation between two gambles was masked by making it seem that the "same" event always gave either a higher or equal outcome under the dominated gamble (the events were colors of marbles drawn from an urn, and the events were not really the same, because the numbers of different colored marbles were not equal in the two urns). Although 58% was not quite significantly different from 50%, it was quite different from the percentage of violations given in another framing of the choice, in which the numbers of marbles of each color were the same in the two urns, and the outcomes for the same events were always higher for the dominant gamble.

Some authors did not consider the evidence of Tversky and Kahneman (1986) convincing, and theories were developed that could account for violations of the Allais paradox without violating stochastic dominance. These rank-dependent theories weaken Savage's independence axiom but preserve coalescing.

RANK-DEPENDENT UTILITY THEORIES

Quiggin (1982, 1985) proposed a rank-dependent utility theory that sparked development of a number of related theories. Quiggin's (1982) original development required that the weight of a probability of 1/2 would be 1/2. However, models were soon proposed that did not impose this requirement. These theories, which weakened the independence axiom, included rank-dependent and rank- and sign-dependent utility theories, including cumulative prospect theory (Lopes, 1990; Luce, 1992; Luce & Fishburn, 1991; 1995; Luce & Narens, 1985; Machina, 1982; Miyamoto, 1989; Schmeidler, 1989; Starmer & Sugden, 1989; Tversky & Kahneman, 1992; Tversky & Wakker, 1995; Wakker, 1989; Wakker, Erev, & Weber, 1994; Wakker & Tversky, 1993; Weber, 1994; Yaari, 1987). These developments were discussed from different perspectives in the book edited by Edwards (1992).

A key property of rank- and sign-dependent utility (RSDU) theories is comonotonic independence, which is either a basic assumption (Wakker & Tversky, 1993) or a consequence of the axiom system (Luce & Fishburn, 1991; 1995).

Comonotonic independence requires that branch independence holds whenever the outcomes maintain the same ranks in the gambles.

Rank-dependent utility (RDU) theories, including cumulative prospect theory (CPT) (Tversky & Kahneman, 1992) and rank- and sign-dependent utility theory (Luce & Fishburn, 1991; 1995), represent the psychological value of a gamble with nonnegative outcomes as follows:

$$\text{RDU}(G) = \Sigma\ [W(P_i) - W(Q_i)]u(x_i) \qquad (5)$$

where RDU(G) is the rank-dependent utility of the gamble, P_i is the (decumulative) probability that an outcome is greater than or equal to x_i; Q_i is the probability that the outcome is strictly greater than x_i. $W(P)$ is a strictly increasing, monotonic function that assigns $W(0) = 0$ and $W(1) = 1$. For three positive outcomes, $0 < x < y < z$, and nonzero probabilities, $p + q + r = 1$, the utility of $G = (x, p; y, q; z, r)$ can be written as follows:

$$\text{RDU}(G) = W(r)u(z) + [W(q + r) - W(r)]u(y) + [1 - W(q + r)]u(x) \qquad (6)$$

With decumulative probability, P, substituted for p and $W(P)$ substituted for $S(p)$ in Equation 3 (with $c = \gamma = .4$), and $u(x) = x^{.4}$, this rank-dependent (or cumulative prospect) theory accounts for the common ratio and common consequence paradoxes without violating stochastic dominance for gambles E and F or G and H. For this model, the predicted certainty equivalents for gambles, A, B, A', B', are \$3,000 > \$432, and \$57 < \$60, respectively; for C, D, C', and D', they are \$500,000 > \$252,525, and \$4,194 < \$7,657, respectively; and for E, F, G, and H, they are \$124 > \$105, and \$113 > \$102.

More generally, Equation 5 (and the special case in Equation 6) must satisfy stochastic dominance and coalescing for all gambles (Birnbaum & Navarrete, 1997; Luce, 1997). When $W(P) = P$, then RDU theory reduces to EU theory. However, when $W(P) \neq P$, the theory implies systematic violations of branch independence. Because the SWU and SWAU theories both satisfy restricted branch independence, the test of "pure" branch independence (apart from the coalescing property that is confounded with it in the Allais common consequence problem) is a test between these two classes of theories.

Restricted Branch Independence

Branch independence was tested in judgments (Birnbaum, Coffey, Mellers, & Weiss, 1992; Weber, Anderson, & Birnbaum, 1992), and systematic violations were found. Such violations are not consistent with SWU. The particular form of branch independence tested in those studies refuted SWU but might be explained by SWAU. Furthermore, such violations, like violations of monotonicity, might occur only in judgment and not also in choice (e.g., Birnbaum & Sutton, 1992). Wakker, Erev, & Weber (1994) tested branch independence in choice and did not find systematic violations of comonotonic or noncomonotonic branch independence. However, their study was designed on the basis of predictions of the model and parameters of Tversky and Kahneman (1992), and their experimental design may have missed gambles that would show violations.

Birnbaum and McIntosh (1996) tested a restricted form of branch independence in choice, in which the probability distributions are the same in all gambles compared. They used a design in which a factorial "net" was cast to check for possible violations in a region of the space of gambles likely to show violations on the basis of Birnbaum, et al. (1992). For three outcome gambles, restricted branch independence can be written as follows:

$$S = (x, p; y, q; z, r) \succ R = (x', p; y', q; z, r)$$

$$\text{if and only if} \tag{7}$$

$$S' = (x, p; y, q; z', r) \succ R' = (x', p; y', q; z', r).$$

where the outcomes are all distinct, and all probabilities are nonzero. Restricted branch independence is implied by both SWU and SWAU models (note that in SWU, the term for the common branch, $S(r)u(z)$, can be subtracted off both sides and replaced with $S(r)u(z')$; in SWAU, the denominators are the same in all four gambles, so both sides can be multiplied by this constant; one can then subtract the common terms and add new common terms, and divide by the common denominator (Birnbaum & Beeghley, 1997).

Constraints on Weighting Function

Birnbaum and McIntosh (1996) tested restricted branch independence with gambles composed of three equally likely outcomes, denoted (x, y, z). They showed that branch independence can be violated in two ways for gambles composed of outcomes selected such that $0 < z < x' < x < y < y' < z'$. The SR' pattern ($S \succ R$ and $S' \prec R'$) occurs if and only if

$$< < \tag{8}$$

where w_L, w_M, and w_H are the weights of the lowest, middle, and highest of three equally likely outcomes, respectively. According to RDU, the weights are as follows: $w_H = W(1/3)$, $w_M = W(2/3) - W(1/3)$, and $w_L = 1 - W(2/3)$.

The RS' pattern of violations, $S \prec R$ and $S' \succ R'$, occurs if and only if

$$> > \tag{9}$$

An experimental tactic employed by Birnbaum and McIntosh was to systematically manipulate both the common outcome, z, and the contrast [(x, y) versus (x', y')] to find outcomes that would be “straddled” by the ratios of weights.

The inverse-S weighting function, used by Tversky and Kahneman (1992) has the property that for three equally likely outcomes, the middle outcome has the least weight. If $w_M < w_L, w_H$ then $> 1 >$, as in Expression 9; therefore, this weighting function combined with CPT implies the RS' pattern of violations. However, empirical choices show the opposite pattern of violations from that predicted by the inverse-S weighting function.

Choices 7 and 8 illustrate these violations of branch independence with gambles in which each outcome has a probability of 1/3.

Choice 7:

S:	1/3 to win \$5	*R*:	1/3 to win \$5
	1/3 to win \$40		1/3 to win \$10
	1/3 to win \$44		1/3 to win \$98

Choice 8:

S':	1/3 to win \$40	*R*':	1/3 to win \$10
	1/3 to win \$44		1/3 to win \$98
	1/3 to win \$107		1/3 to win \$107

Birnbaum and McIntosh (1996) found that most subjects preferred *S* to *R* but most subjects preferred *R*' to *S*'. In all twelve variations examined, the frequency of the *SR*' pattern of violations was greater than the frequency of *RS*' choices. A similar pattern of violations of restricted branch independence was also observed by Birnbaum and Chavez (1997) and Birnbaum and Navarrete (1997), who used choices between gambles with unequal probabilities (but the same in each gamble compared).

Similar (but distinct) violations of branch independence were observed in judgments of buying and selling prices of three and four outcome gambles by Birnbaum and Beeghley (1997) and Birnbaum and Veira (1998).

Problems for the Inverse-S Weighting Function

The pattern of violations of branch independence found in all of these studies [$S \succ R$ and $S' \prec R'$] is opposite that predicted from the inverse-S weighting function, $W(P)$, estimated by Tversky and Kahneman (1992) and Wu and Gonzalez (1996). Either the W(P) function changed between studies, or something is wrong with the RDU models.

This *SR*' pattern can be fit by RDU, with $u(x) = x$, with $W(1/3) = .16$, $W(2/3) = .49$. The pattern is consistent with Expression 8 rather than Expression 9. The *SR*' pattern is not consistent with any inverse-S weighting function in which the weight of the middle outcome is least.

Birnbaum and McIntosh (1996) interpreted the contradiction as evidence of a configural weighting model, which is equivalent to the RDU model in the experiment of Birnbaum and McIntosh (1996), but which can be tested against RDU in other experiments.

In summary, violations of restricted branch independence rule out SWU and SWAU models, but they can be explained by RDU models. However, the weighting function used by RDU to explain the violations of branch independence is quite different from that used to explain the Allais paradoxes. This apparent contradiction in the weighting function does not pose a problem, however, for configural weight models.

CONFIGURAL WEIGHT THEORY

Configural weighting models were proposed by Birnbaum, Parducci, & Gifford (1971), Birnbaum (1973; 1974) and Birnbaum and Stegner (1979; 1981) to account for violations of additive independence in psychophysical and evaluative judgments. Shanteau (1974; 1975) observed similar violations of the additive model in judgments of risky gambles. Configural weight models are similar to RDU in that the weight of a stimulus can be affected by the rank of the stimulus in the configuration of stimuli to be combined. They do not, however, impose the "pure" rank-dependence of RDU (Equation 5) that requires stochastic dominance. The models are configurally weighted averages, and like SWAU, they imply violations of coalescing and stochastic dominance; however, like RDU they predict violations of branch independence.

To compare various configural models, it will be helpful to introduce a brief taxonomy. The configurally weighted, average configural value model can be written as follows:

$$\text{CWACV}(G) = \sum w(x_i, G)u(x_i, G)/\sum w(x_i, G) \tag{10}$$

where $w(x_i, G)$ and $u(x_i, G)$ are the weight and utility of outcome x_1 in gamble G.

If $w(x_i, G) = w(p_i)$ in Equation 10, the model is termed a *configural value* model; if $w(p) = p$, this model reduces to lottery-dependent utility (Becker & Sarin, 1987; Currim & Sarin, 1992; Daniels & Keller, 1992). Previous investigations of lottery-dependent utility have further restricted the lottery-dependent utility models to ensure stochastic dominance.

When $u(x_i, G) = u(x_i)$, Equation 10 is termed a weighted averaging model. If $w(x_i, G) = w(x_i, p_i)$, and $u(x_i, G) = u(x_i)$, the model is termed a *differentially weighted averaging* model. A special case of differential weighting is constant weighting, also called SWAU, where $w(x_i, G) = w(x_i, p_i) = w(p_i)$. Constant weight and differentially weighted models have not proved as successful in experimental tests of judgment as *configurally weighted* models in which the weights of the outcomes are affected by their relative positions in the gamble rather than by their values (Birnbaum, 1973; 1974; Birnbaum & Stegner, 1979).

A special case of such configural weighting is RDU, discussed earlier, in which the configural weights depend on a functional of decumulative probability. Two other configural weight models in which weights are affected by the ranks of the outcomes, the RAM model and TAX model, are discussed in the next section.

Rank Affected Multiplicative Configural Weighting

A configural weighting model in which weights are the product of a function of the rank of the outcome and a function of the probability of the outcome will be termed the Rank Affected Multiplicative (RAM) model. This model can be written as follows:

$$w(x_i, G) = a(V, r_i, s_i, n)S(p_i) \tag{11}$$

where the weight of outcome x_i in Gamble G depends on the product of a function of probability, $S(p)$, and a configural weight that depends on the judge's viewpoint (V), the rank of the outcome among the other outcomes (here rank depends on the values but not the probabilities of the outcomes; rank is counted from $r_1 = 1$ = highest, to r_n = n = lowest outcome; s_i is the augmented sign of outcome x_i (it takes on the levels, –, 0, and +); and n is the number of outcomes in the gamble. If the experiment is restricted to a single viewpoint (e.g., choice), all positive outcomes, and three-outcome gambles, the model has three values of a, of which one can be fixed (Birnbaum, 1997).

Birnbaum and McIntosh (1996) estimated the values of a to be .51, .33, and .16 for lowest, middle, and highest of three equally likely outcomes in a choice experiment. To fit the Tversky and Kahneman (1992) data, Birnbaum and McIntosh (1996) estimated a = .63 and .37 for lowest and highest of two positive outcomes, and $S(p) = p^{.6}$. The Birnbaum and McIntosh (1996) model also used the approximation, $u(x) = x$, for $0 < x < \$150$. The same model was fit by Birnbaum and Beeghley (1997) to judgments of the buying prices (what is the most a buyer should pay to purchase the gamble?) and selling prices (what is the least that a seller should accept to sell the gamble, rather than play it?). Birnbaum and Beeghley (1997) found that in the buyer's viewpoint, the values of a were .56, .36, and .08 for the lowest, middle, and highest of three equally likely outcomes; from the seller's viewpoint, the values were .27, .52, and .21. [These parameter estimates are for group data; however, they are also representative of individual subjects. Information on individual subject parameters is given in Birnbaum and McIntosh (1996), Birnbaum and Beeghley (1997), and Birnbaum and Chavez (1997).]

Configural Weight, TAX Model

Birnbaum and Stegner (1979) had considered a different configural weight model that is equivalent to the RAM model fit by Birnbaum and McIntosh (1996) when the experiment uses a fixed probability distribution and a fixed number of outcomes (e.g., as in Birnbaum & McIntosh), but which makes different predictions when the number and probabilities of common outcomes is manipulated. The Birnbaum and Stegner "revised" model assumes that weight is transferred among stimuli according to the ranks of the utilities of the outcomes in proportion to the weight of the stimulus that is losing weight. This model will be termed the *tax* model to indicate that the weight transferred is a proportion of the weight to be reduced. The weight TAX model violates asymptotic independence and can violate distribution independence (Birnbaum, 1997), unlike the multiplicative viewpoint by probability model. However, both configural weight models can explain violations of cumulative independence and stochastic dominance.

This TAX model can be written for positive outcomes as follows:

$$U(G) = \qquad (12)$$

where $\omega(i, j, G)$ is the configural weight transferred from the lower outcome j to a higher outcome i.

A simplifying assumption that gave a reasonable fit to the experiment of Birnbaum and Chavez (1997) is as follows:

$$\omega(i, j, G) = \delta S(p_i)/(n + 1) \text{ if } \delta < 0 \quad (13a)$$

$$\omega(i, j, G) = \delta S(p_j)/(n + 1) \text{ if } \delta \geq 0 \quad (13b)$$

where δ is the single configural parameter. If $\delta < 0$, weight is transferred from a higher outcome to a lower outcome as an increasing function of the probability of the higher outcome. If $\delta = -1$, this model yields configural weights of 2/3 and 1/3, for the lower and higher of two equally likely outcomes, 3/6, 2/6, and 1/6 for lowest, middle, and highest of three equally likely outcomes, and 4/10, 3/10, 2/10, and 1/10 for the lowest to highest of four equally likely outcomes.

Both of these configural models imply violations of cumulative independence and stochastic dominance, unlike RDU theories. The multiplicative, RAM model (Eq. 11) also implies distribution independence, unlike the TAX model.

CUMULATIVE INDEPENDENCE AND STOCHASTIC DOMINANCE

Rank- and sign-dependent utility theories imply two cumulative independence conditions derived by Birnbaum (1997).

Gambles are selected such that $0 < z < x' < x < y < y' < z'$ and $p + q + r = 1$.

Lower Cumulative Independence:

If $S = (z, r; x, p; y, q) \succ R = (z, r; x', p; y', q)$

Then $S'' = (x', r; y, p + q) \succ R'' = (x', r + p; y', q)$ (14)

Upper Cumulative Independence:

If $S' = (x, p; y, q; z', r) \prec R' = (x', p; y', q; z', r)$

Then $S''' = (x, p + q; y', r) \prec R''' = (x', p; y', q + r)$ (15)

Any theory that satisfies comonotonic branch independence, monotonicity, transitivity, and coalescing must satisfy both lower and upper cumulative independence (Birnbaum, 1997; Birnbaum & Navarrete, 1997). Thus, RSDU and CPT, which reduce to RDU in the domain of gains both imply cumulative independence.

Violations of Cumulative Independence

Birnbaum and Navarrete (1997) tested 27 variations of lower and upper cumulative independence and branch independence, using different probability distributions and different values of the outcomes. One such test of lower cumulative independence is illustrated in Choices 9 and 10.

Choice 9:

S:	.8 probability to win \$3	*R*:	.8 probability to win \$3
	.1 probability to win \$48		.1 probability to win \$10
	.1 probability to win \$52		.1 probability to win \$98

Choice 10:

S'':	.8 probability to win \$10	*R''*:	.9 probability to win \$10
	.2 probability to win \$52		.1 probability to win \$98

Most subjects chose *S* over *R* in Choice 9; however most subjects preferred *R''* over *S''* in Choice 10. Overall, tests of lower cumulative independence found that the majority of judges showed more choices in the *SR''* pattern ($S \succ R$ and $R'' \succ S''$), which violates lower cumulative independence, than in the *RS''* pattern, which would be consistent with it.

Upper cumulative independence was also systematically violated, as illustrated in Choices 11 and 12.

Choice 11:

S':	.1 probability to win \$40	*R'*:	.1 probability to win \$10
	.1 probability to win \$44		.1 probability to win \$98
	.8 probability to win \$110		.8 probability to win \$110

Choice 12:

S''':	.2 probability to win \$40	*R'''*:	.1 probability to win \$10
	.8 probability to win \$98		.9 probability to win \$98

Most subjects chose *R'* over *S'* in Choice 11; however, most subjects chose *S'''* over *R'''* in Choice 12. Overall, there were more subjects who had more choices in the order *R'S'''*, which violates upper cumulative independence than in the order, *S'R'''*, which would be consistent with it.

Such systematic violations of cumulative independence are inconsistent with RDU theories, including RSDU and CPT. These theories also fail to predict systematic violations of stochastic dominance.

Violations of Stochastic Dominance

Birnbaum (1997) noted that the model of Birnbaum and McIntosh (1996) predicts violations of stochastic dominance in choices between three-outcome gambles generated from the following recipe. Start with a two outcome gamble, $G^0 = (x, p; y, q)$, where $0 < x < y$ and $p + q = 1$. Create a strictly worse, three-outcome gamble by splitting the branch with the higher outcome, where the new outcome ($y^–$) is slightly worse than y: $G^– = (x, p; y^–, r; y, q – r)$. Then create a strictly better gamble, G^+ by splitting the branch in G^0 with the lowest outcome, where the new outcome (x^+) is slightly better than x: $G^+ = (x, p \; – r; x^+, r; y, q)$. Choice 13 illustrates an example of this recipe in which G^+ dominates $G^–$.

Choice 13:

G^+:	.05 probability to win $12	$G^–$:	.10 probability to win $12
	.05 probability to win $14		.05 probability to win $90
	.90 probability to win $96		.85 probability to win $96

Most judges (73% in this case) chose the dominated gamble ($G^–$) over the dominant gamble (G^+) in direct choice. Similar results were obtained with other choices constructed from this recipe (Birnbaum & Navarrete, 1997).

This violation of stochastic dominance could result from a violation of transitivity, monotonicity, or coalescing. The property that seems most likely to be crucial is coalescing. Imagine the following Gedanken experiment: Suppose the gambles above were presented as four-outcome gambles with all events split, as in Savage's representation of the Allais paradox. It seems quite unlikely that judges would select the split version of $G^–$, ($12, .05; $12, .05; $90; .05; $96, .85) over the split version of G^+, ($12, .05; $14, .05, $96, .05; $96, .85).

Coalescing and Event Splitting Effects

It seems unlikely that judges would fail to recognize coalescing in a direct test. For example, they should easily recognize that ($12, .1; $96, .9) is the same as ($12, .05; $12, .05; $96, .9). However, when gambles are compared indirectly by comparing their choices against a third gamble, the combination of coalescing and transitivity has been violated. Starmer and Sugden (1993) and Humphrey (1995) found such violations of coalescing, called "event-splitting" effects.

Suppose that S_2 is the split version and S_1 is the coalesced version of the same gamble. Similarly, let R_1 be the coalesced version of R_2. Coalescing implies that $S_2 \sim S_1$ and $R_1 \sim R_2$; therefore, by transitivity, $S_1 \succ R_1$ if and only if $S_2 \succ R_2$.

Choices 14 and 15 are from Humphrey (1995), except outcomes are in dollars instead of pounds.

Choice 14:

S_1:	.7 probability to win \$8	R_1:	.3 probability to win \$24
	.3 probability to win \$0		.7 probability to win \$0

Choice 15:

S_2:	.3 probability to win \$8	R_2:	.3 probability to win \$24
	.4 probability to win \$8		.4 probability to win \$0
	.3 probability to win \$0		.3 probability to win \$0

Choice 14 is the same as Choice 15 if coalescing and transitivity hold. Humphrey (1995) found that more subjects had the order $R_1 \succ S_1$ and $S_2 \succ R_2$, than the opposite, as if the split branch has more weight. Starmer and Sugden (1993) and Humphrey (1995) noted that their results were inconsistent with the editing principle of combination, and they interpreted their results as consistent with a SWU model. Violations of coalescing are also inconsistent with RDU models, including CPT with or without the editing principle.

Event-Splitting Independence

The SWU model implies event-splitting independence (Birnbaum & Navarrete, 1997). Event-splitting independence assumes that if a branch with a positive outcome is split, the effect of splitting an event should be independent of the relative position within the gamble of the outcome associated with that event. SWU models imply event-splitting independence, but averaging models do not.

In averaging models, including SWAU and the configural weight models, when $S(p + q) < S(p) + S(q)$, splitting a branch with a positive outcome can either increase or decrease the value of a gamble, depending on the whether the outcome split was the highest or lowest outcome in the gamble, respectively. Judgment data collected in collaboration with Sherry Yeary and Teresa Martin suggest that event splitting independence, cumulative independence, and stochastic dominance are all violated; however, event-splitting independence has not yet been tested in choice.

Violations of Distribution Independence

Distribution independence asserts that preference should be independent of the probability distribution of common branches (Birnbaum & Chavez, 1997). For four-outcome gambles, distribution independence can be written as follows:

$$\mathrm{S} = (x, p; y, q; z; r; v, s) \succ R = (x', p; y', q; z; r; v, s)$$

$$\text{if and only if} \qquad (16)$$

$$\mathrm{S}' = (x, p; y, q; z; r'; v, s') \succ R' = (x', p; y', q; z; r'; v, s')$$

where $s = 1 - p - q - r$ and $s' = 1 - p - q - r'$. As contrasted with branch independence, distribution independence assumes that the probabilities of common outcomes should have no effect on the choice, whereas branch independence assumes that holding the probabilities fixed, the outcomes on the common branches should have no effect on the choice. If coalescing and transitivity are assumed, then distribution independence follows from branch independence.

EU, SWU, SWAU, OP, and the RAM model used by Birnbaum & McIntosh (1996) all imply distribution independence, when $0 < z < x' < x < y < y' < v$, and all probabilities are positive. RDU and the configural weight, TAX model of Birnbaum & Stegner (1979) violate distribution independence.

An example problem from Birnbaum and Chavez (1997) testing distribution independence is given in Choices 16 and 17.

Choice 16:

S:	.59 probability to win \$4	*R*:	.59 probability to win \$4
	.20 probability to win \$45		.20 probability to win \$11
	.20 probability to win \$49		.20 probability to win \$97
	.01 probability to win \$110		.01 probability to win \$110

Choice 17:

S':	.01 probability to win \$4	*R'*:	.01 probability to win \$4
	.20 probability to win \$45		.20 probability to win \$11
	.20 probability to win \$49		.20 probability to win \$97
	.59 probability to win \$110		.59 probability to win \$110

Note that Choices 16 and 17 have common branches that if “trimmed” before comparison would leave the same contrast. Birnbaum and Chavez (1997) found systematic violations of distribution independence. More subjects had the preference pattern *SR'* than the pattern *RS'*. This result was observed for all 12 variations of the above choices. Birnbaum and Chavez also found systematic violations of branch independence.

Furthermore, violations of distribution independence and branch independence were compatible with each other, according to either RDU or the weight tax model. Branch independence and distribution independence are ruled out by the editing principles of OP. Distribution independence is implied by original prospect theory with or without the editing principles. Thus, violations of distribution independence rule out not only EU and the editing principle of cancellation, but also OP and the RAM model.

The violations of branch independence and distribution independence were consistent with the findings of Birnbaum and McIntosh (1996) and inconsistent with the inverse-S weighting function used in CPT to account for the Allais paradoxes.

SUMMARY AND CONCLUSIONS

In summary, the paradoxes of Allais refute EU theory, but they can be explained by a number of models proposed to account for them. These models make different predictions for a series of new independence conditions that can test among rival theories. Table 1 shows that two properties, branch independence and coalescing segregate the models into four categories: EU and EV models satisfy both properties; SWU and SWAU violate coalescing but satisfy restricted branch independence; RDU violates branch independence, but satisfies coalescing; and CWU theories violate both of these properties.

Table 1. Two Properties that Test among Decision Theories

	Branch Independence	
Coalescing	Satisfied	Violated
Satisfied	EU	RDU
Violated	SWU	CWU

Theories with editing principles, OP and CPT, are more difficult to place in the table. Without the editing principles, OP is in the same category as SWU, and CPT is in the same category as RDU. The editing principle of cancellation implies no violations of branch independence or distribution independence, and the editing principle of combination implies coalescing. CPT satisfies coalescing with or without the editing principles.

Evidence shows systematic violations of restricted branch independence in both judgment and choice (Birnbaum & Beeghley, 1997; Birnbaum & Chavez, 1997; Birnbaum & McIntosh, 1996; Birnbaum & Navarrete, 1997; Birnbaum & Veira, 1998). Systematic violations of restricted branch independence are inconsistent with EU, SWU, and SWAU models. They also rule out the editing principle of cancellation as a theory of what people do when confronted with common branches in choice problems.

Problems for RDU and RSDU

Systematic violations of branch independence are consistent with the RDU models. If CPT drops the editing principle of cancellation, then the representation of RSDU used by CPT can explain violations of branch independence. However, the pattern of violations observed is opposite that predicted by the inverse-S weighting function used in CPT to account for certainty equivalents of binary gambles (Tversky & Kahneman, 1992) and to account for violations of Allais independence (Wu & Gonzalez, 1996).

This contradiction within CPT between the Allais paradox and restricted branch independence can be tested within a single study by the two properties of cumulative independence. Both cumulative independence conditions appear to be systematically violated by empirical choices (Birnbaum & Navarrete, 1997). Furthermore, choices systematically violate stochastic dominance in the manner predicted by configural weight models. Violations of cumulative independence and stochastic dominance appear to be due to violations of coalescing, a conclusion that is also consistent with research on event splitting effects (Starmer & Sugden, 1993; Humphrey, 1995). However, because cumulative independence and stochastic dominance are combinations of simpler properties, further research on the property of coalescing is needed, especially to test the predicted violations of event-splitting independence implied by configural weighting models.

These results suggest that there are two separate causes of the Allais paradox: subjects violate both branch independence and coalescing. Both the multiplicative form of configural weighting of the RAM model and the TAX model of Birnbaum and Stegner (1979) as modified by Birnbaum and Chavez (1997) can account for violations of branch independence and coalescing. Both of these models can explain violations of cumulative independence and stochastic dominance. The RAM model cannot account for violations of distribution independence, however, which the TAX model can.

All of the models except the configural weight TAX model are inconsistent with one or more of the experiments reviewed here. Although SWU and SWAU can account for the Allais paradoxes, they fail to predict violations of branch independence or distribution independence. Original prospect theory, with the editing principle of cancellation implies no violations of branch independence, and with or without the editing principle, it predicts no violations of distribution independence. RDU and RSDU, including CPT, imply no violations of stochastic dominance, no event-splitting effects, and no violations of cumulative independence. The RAM model used by Birnbaum and McIntosh (1996) can account for all of the phenomena except violations of distribution independence.

TAX Model Account of the Phenomena

The configural weight, TAX model of Equations 12 and 13 can account for all of the results reviewed here with the same parameters. Although the model allows a nonlinear $u(x)$ function, it is possible for this model to account for all of the choices reviewed here with the assumption that $u(x) = x$. Suppose that $S(p) = p^{.7}$, and that $\delta = -1$. Equations 12 and 13 then yield the following predicted certainty equivalents for the gambles:

For Choices 1 and 2, $U(A)$ = \$3,000 > $U(B)$ = \$1,934 and $U(A')$ = \$633 < $U(B')$ = \$733, thus accounting for the common ratio effect. The model predicts the common consequence paradox in Choices 3 and 4, because $U(C)$ = \$500,000 > $U(D)$ = \$405,106, and $U(C')$ = \$62,643 < $U(D')$ = \$117,879.

The TAX model correctly predicts satisfaction of stochastic dominance in the transparent Choices 5 and 6, $U(E)$ = \$133 > $U(F)$ = \$103 and $U(G)$ = \$113 > $U(H)$ = \$102. The model accounts for violations of restricted branch independence in Choices 7 and 8, $U(S)$ = \$23.17 > $U(R)$ = \$22.16 and $U(S')$ = \$52.49 < $U(R')$ = \$55.51. For violations of lower cumulative independence in Choices 9 and 10, $U(S)$ = \$14.05 > $U(R)$ = \$11.67 and $U(S'')$ = \$17.69 < $U(R'')$ = \$20.37. Violations of upper cumulative independence in Choices 11 and 12 agree with the predictions: $U(S')$ = \$65.03 < $U(R')$ = \$69.59 and $U(S''')$ = \$68.04 > $U(R''')$ = \$58.29. Although the model satisfies stochastic dominance in the obvious cases of Choices 5 and 6, it correctly predicts violations in Choice 13: $U(G^+)$ = \$45.77 < $U(G^-)$ = \$63.10.

The configural weight, TAX model accounts for violations of coalescing (event-splitting effects) in Choices 14 and 15, $U(S_1)$ = \$3.44 < $U(R_1)$ = \$5.69 and $U(S_2)$ =

\$4.14 > $U(R_2)$ = \$3.72. It also explains the violations of distribution independence in Choices 16 and 17, since $U(S)$ = \$21.70 > $U(R)$ = \$20.56 and $U(S')$ = \$49.85 < $U(R')$ = \$50.03. In summary, this model accounts for the following phenomena with the same parameters: the common ratio and common consequence paradoxes of Allais, violations of branch independence, violations of lower and upper cumulative independence, violations of distribution independence, violations of coalescing (event splitting effects), and cases where stochastic dominance is satisfied and violated by empirical choices.

The original question posed by Edwards (1962) thirty-five years ago is still relevant to theorists in behavioral decision making: how many pages are there in the book of weights? The answer will depend on the theory in which the weights operate. Evidence reviewed here suggests that the theory that requires the shortest book to account for existing data is the configural weight, TAX model. For positive outcomes, this model would require a single page of $S(p)$ weights and a single page describing how the configural parameter, δ, depends on the subject's point of view. Perhaps there is only one configural parameter, δ, for the viewpoint of choice. Such a model remains standing as a viable null hypothesis for future research. It seems unlikely that this model, or any model, can remain standing in the face of empirical data as long as Edwards' original, and clear statement of the issues facing behavioral decision theory.

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REFERENCES

Allais, M. (1953). Le comportement de l'homme rationnel devant le risque: Critique des postulats et axiomes de l'école Américaine. *Econometrica*, *21*, 503-546.

Allais, M. (1979). The foundations of a positive theory of choice involving risk and a criticism of the postulates and axioms of the American School. In M. Allais & O. Hagen (Eds.), *Expected utility hypothesis and the Allais paradox* (pp. 27-145). Dordrecht, The Netherlands: Reidel.

Allais, M., & Hagen, O. (Eds.). (1979). *Expected utility hypothesis and the Allais paradox*. Dordrecht, The Netherlands: Reidel.
Becker, J., & Sarin, R. (1987). Lottery dependent utility. *Management Science*, *33*, 1367-1382.
Birnbaum, M. H. (1973). Morality judgment: Test of an averaging model with differential weights. *Journal of Experimental Psychology*, *99*, 395-399.
Birnbaum, M. H. (1974). The nonadditivity of personality impressions. *Journal of Experimental Psychology*, *102*, 543-561.
Birnbaum, M. H. (1997). Violations of monotonicity in judgment and decision making. In A. A. J. Marley (Ed.), *Choice, decision and measurement: Essays in honor of R. Duncan Luce* (pp. 73-100). Mahwah, NJ: Erlbaum.
Birnbaum, M. H., & Beeghley, D. (1997). Violations of branch independence in judgments of the value of gambles. *Psychological Science*, *8*, 87-94.
Birnbaum, M. H., & Chavez, A. (1997). Tests of Theories of Decision Making: Violations of Branch Independence and Distribution Independence. *Organizational Behavior and Human Decision Processes, 71*, 161-194.
Birnbaum, M. H., Coffey, G., Mellers, B. A., & Weiss, R. (1992). Utility measurement: Configural-weight theory and the judge's point of view. *Journal of Experimental Psychology: Human Perception and Performance*, *18*, 331-346.
Birnbaum, M. H., & McIntosh, W. R. (1996). Violations of branch independence in choices between gambles. *Organizational Behavior and Human Decision Processes*, *67*, 91-110.
Birnbaum, M. H., & Navarrete, J. B. (1997). Testing rank- and sign-dependent utility theories: Violations of stochastic dominance and cumulative independence. Unpublished manuscript.
Birnbaum, M. H., Parducci, A., & Gifford, R. K. (1971). Contextual effects in information integration. *Journal of Experimental Psychology*, *88*, 158-170.
Birnbaum, M. H., & Stegner, S. E. (1979). Source credibility in social judgment: Bias, expertise, and the judge's point of view. *Journal of Personality and Social Psychology*, *37*, 48-74.
Birnbaum, M. H., & Stegner, S. E. (1981). Measuring the importance of cues in judgment for individuals: Subjective theories of IQ as a function of heredity and environment. *Journal of Experimental Social Psychology*, *17*, 159-182.
Birnbaum, M. H., & Sutton, S. E. (1992). Scale convergence and utility measurement. *Organizational Behavior and Human Decision Processes*, *52*, 183-215.
Birnbaum, M. H., & Veira, R. (1998). Configural weighting in judgments of two- and four-outcome gambles. *Journal of Experimental Psychology: Human Perception and Performance*, *24*, 216-226.
Camerer, C. F. (1989). An experimental test of several generalized utility theories. *Journal of Risk and Uncertainty*, *2*, 61-104.
Currim, I. S., & Sarin, R. K. (1992). Robustness of expected utility model in predicting individual choices. *Organizational Behavior and Human Decision Processes*, *52*, 544-568.
Daniels, R. L., & Keller, L. R. (1992). Choice-based assessment of utility functions. *Organizational Behavior and Human Decision Process*, *52*, 524-543.
Edwards, W. (1954). The theory of decision making. *Psychological Bulletin*, *51*, 380-417.

Edwards, W. (1962). Subjective probabilities inferred from decisions. *Psychological Review*, *69*, 109-135.
Edwards, W. (Ed.). (1992). *Utility theories: Measurements and applications*. Boston, MA: Kluwer Academic Publishers.
Edwards, W. (1992). Towards the demise of economic man and woman: Bottom lines from Santa Cruz. In W. Edwards (Ed.), *Utility theories: Measurements and applications* (pp. 254-267). Boston, MA: Kluwer Academic Publishers.
Fishburn, P. C. (1978). On Handa's "New theory of cardinal utility" and the maximization of expected return. *Journal of Political Economy*, *86*(2), 321-324.
Humphrey, S. J. (1995). Regret aversion or event-splitting effects? More evidence under risk and uncertainty. *Journal of risk and uncertainty*, *11*, 263-274.
Kahneman, D., & Tversky, A. (1979). Prospect theory: An analysis of decision under risk. *Econometrica*, *47*, 263-291.
Karmarkar, U. S. (1978). Subjectively weighted utility: A descriptive extension of the expected utility model. *Organizational Behavior and Human Performance*, *21*, 61-72.
Karmarkar, U. S. (1979). Subjectively weighted utility and the Allais paradox. *Organizational Behavior and Human Performance*, *24*, 67-72.
Keller, L. R. (1985). The effects of problem representation on the sure-thing and substitution principles. *Management Science*, *31*, 738-751.
Lattimore, P. K., Baker, J. R., & Witte, A. D. (1992). The influence of probability on risky choice. *Journal of Economic Behavior and Organization*, *17*, 377-400.
Lopes, L. (1990). Re-modeling risk aversion: A comparison of Bernoullian and rank dependent value approaches. In G. M. v. Furstenberg (Eds.), *Acting under uncertainty* (pp. 267-299). Boston: Kluwer.
Luce, R. D. (1997). Coalescing, event commutativity, and theories of utility. Unpublished manuscript.
Luce, R. D. (1992). Where does subjective expected utility fail descriptively? *Journal of Risk and Uncertainty*, 5, 5-27.
Luce, R. D., & Fishburn, P. C. (1991). Rank- and sign-dependent linear utility models for finite first order gambles. *Journal of Risk and Uncertainty*, *4*, 29-59.
Luce, R. D., & Fishburn, P. C. (1995). A note on deriving rank-dependent utility using additive joint receipts. *Journal of Risk and Uncertainty, 11*, 5-16.
Luce, R. D., & Narens, L. (1985). Classification of concatenation measurement structures according to scale type. *Journal of Mathematical Psychology*, *29*, 1-72.
Machina, M. J. (1982). Expected utility analysis without the independence axiom. *Econometrica*, *50*, 277-323.
Miyamoto, J. M. (1989). Generic utility theory: measurement foundations and applications in multiattribute utility theory. *Journal of Mathematical Psychology*, *32*, 357-404.
Preston, M. G., & Baratta, P. (1948). An experimental study of the auction-value of an uncertain outcome. *American Journal of Psychology*, *61*, 183-193.
Quiggin, J. (1982). A theory of anticipated utility. *Journal of Economic Behavior and Organization, 3*, 324-345.
Quiggin, J. (1985). Subjective utility, anticipated utility, and the Allais paradox. *Organizational Behavior and Human Decision Processes*, *35*, 94-101.
Savage, L. J. (1954/1972). *The foundations of statistics* (second revised edition). New York: Dover Publications, Inc.

Schoemaker, P. J. (1982). The expected utility model: Its variants, purposes, evidence and limitations. *Journal of Economic Literature*, *20*, 529-563.

Schmeidler, D. (1989). Subjective probability and expected utility without additivity. *Econometrica*, *57*, 571-587.

Shanteau, J. (1974). Component processes in risky decision making. *Journal of Experimental Psychology*, *103*, 680-691.

Shanteau, J. (1975). Information integration analysis of risky decision making. In M. Kaplan & S. Schwartz (Eds.), *Human judgment and decision processes* (pp. 109-137). New York: Academic Press.

Slovic, P., & Tversky, A. (1974). Who accepts Savage's axiom? *Behavioral Science*, *19*, 368-373.

Starmer, C. (1992). Testing new theories of choice under uncertainty using the common consequence effect. *Review of Economic Studies*, *59*, 813-830.

Starmer, C., & Sugden, R. (1989). Violations of the independence axiom in common ratio problems: An experimental test of some competing hypotheses. *Annals of Operations Research*, *19*, 79-101.

Starmer, C., & Sugden, R. (1993). Testing for juxtaposition and event-splitting effects. *Journal of risk and uncertainty*, *6*, 235-254.

Stevenson, M. K., Busemeyer, J. R., & Naylor, J. C. (1991). Judgment and decision-making theory. In M. Dunnette & L. M. Hough (Eds.), *New handbook of industrial-organizational psychology* (pp. 283-374). Palo Alto, CA: Consulting Psychologist Press.

Tversky, A., & Kahneman, D. (1986). Rational choice and the framing of decisions. *Journal of Business*, *59*, S251-S278.

Tversky, A., & Kahneman, D. (1992). Advances in prospect theory: Cumulative representation of uncertainty. *Journal of Risk and Uncertainty*, *5*, 297-323.

Tversky, A., & Wakker, P. (1995). Risk attitudes and decision weights. *Econometrica*, *63*, 1255-1280.

Viscusi, K. W. (1989). Prospective reference theory: Toward an explanation of the paradoxes. *Journal of risk and uncertainty*, *2*, 235-264.

von Winterfeldt, D., & Edwards, W. (1986). *Decision analysis and behavioral research*. Cambridge, England: Cambridge University Press.

Wakker, P. (1989). Transforming probabilities without violating stochastic dominance. In E. E. Roskam (Eds.), *Mathematical psychology in progress* (pp. 29-47). Berlin: Springer.

Wakker, P., Erev, I., & Weber, E. U. (1994). Comonotonic independence: The critical test between classical and rank-dependent utility theories. *Journal of Risk and Uncertainty*, *9*, 195-230.

Wakker, P., & Tversky, A. (1993). An axiomatization of cumulative prospect theory. *Journal of Risk and Uncertainty*, *7*, 147-176.

Weber, E. U. (1994). From subjective probabilities to decision weights: The effects of asymmetric loss functions on the evaluation of uncertain outcomes and events. *Psychological Bulletin*, *114*, 228-242.

Weber, E. U., Anderson, C. J., & Birnbaum, M. H. (1992). A theory of perceived risk and attractiveness. *Organizational Behavior and Human Decision Processes*, *52*, 492-523.

Wu, G., & Gonzalez, R. (1996). Curvature of the probability weighting function. *Management Science*, *42*, 1676-1690.

Yaari, M. E. (1987). The dual theory of choice under risk. *Econometrica*, *55*, 95-115.